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A PROBABILISTIC ANALYSIS OF A CATASTROPHIC TRANSURANIC WASTE HOIST ACCIDENT AT THE WIPP

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PREFACE

This report builds upon the extensive and careful analyses made by the DOE of the probability of failure of the waste hoist, and more particularly on the probability of failure of a major component, the hydraulic brake system. The extensive fault tree analysis prepared by the DOE was the starting point of the present report. A key element of this work is the use of probability distributions rather than so-called point estimates to describe the probability of failure of an element. One of the authors (MAG) developed the expressions for the probability of failure of the brake system. The second author (TJS) executed the calculations of the final expressions for failure probabilities.

The authors hope that this work will be of use to the DOE in its evaluation of the safety of the waste hoist, a key element at the WIPP.

FOREWORD

The purpose of the Environmental Evaluation Group (EEG) is to conduct an independent technical evaluation of the Waste Isolation Pilot Plant (WIPP) Project to ensure protection of the public health and safety and the environment. The WIPP Project, located in southeastern New Mexico, is being constructed as a repository for permanent disposal of transuranic (TRU) radioactive wastes generated by the national defense programs. The EEG was established in 1978 with funds provided by the U.S. Department of Energy (DOE) to the State of New Mexico. Public Law 100-456, the National Defense Authorization Act, Fiscal Year 1989, Section 1433, assigned EEG to the New Mexico Institute of Mining and Technology and continued the original contract DE-AC04-79AL10752 through DOE contract DE-AC04-89AL58309.

The New Mexico Radioactive and Hazardous Materials Act, Laws 1991, Ch. 2, Section 2, defined EEG as the independent state review facility for the impact on health and safety of the WIPP. The 1992 WIPP Land Withdrawal Act, Public Law 102-579, requires the DOE to consult and cooperate with the EEG.

EEG performs independent technical analyses of the suitability of the proposed site; the design of the repository, its planned operation, and its long-term integrity; suitability and safety of the transportation systems; suitability of the Waste Acceptance Criteria and the generator sites' compliance with them; and related subjects. These analyses include assessments of reports issued by the DOE and its contractors, other federal agencies and organizations, as they relate to the potential health, safety and environmental impacts from WIPP. Another important function of EEG is independent environmental monitoring of background radioactivity in air, water, and soil, both on-site and in surrounding communities.

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SUMMARY

An assessment is made of the various DOE reports describing the safety of the waste hoist at WIPP. The DOE reports include studies by Banz et al. (1985), an unpublished draft by Chan et al. (1987), and more recently the Final Safety Analysis Report (FSAR), May 1990, Volume III, Chapter 7, Appendix 7B. The most definitive DOE study appears in the IRA DOE/WIPP-89-010, 1990, WIPP Integrated Risk Assessment (IRA), Volume II, Section 4.3. The earlier studies identified the possible failure of the hydraulic brake system as the most important contributor in the accident scenarios. As a consequence the WIPP IRA includes an analysis of the likelihood of an accident to the braking system, based on a probabilistic risk assessment.

This report accepts as a given the design of the hydraulic brake system, and the elaborate and complete fault tree analysis appearing in the WIPP IRA of the possible failure modes of the brake system. This report does assess two aspects of the DOE report which present difficulties. The first relates to a major DOE decision (FSAR May 1990, p. 7B-2) to describe all results in terms of so-called "point estimates," rather than including probability distributions, which provide estimates of uncertainties. The DOE bases its decision, in part, on a recommendation of the NRC, appearing in the Federal Register, to use mean calculated values of individual risk to compare with quantitative safety goals. However, a close reading of the reference in the Federal Register reveals that in addition to recommending the use of mean calculated values, the NRC also recommends the use, where practicable, of estimates of uncertainties. This makes it possible to estimate the confidence level to be ascribed to quantitative results.

A second aspect of the DOE reports is the reliance on data sources that, in some instances, are not the most recently available.

This report describes a methodology for calculating probability distributions to evaluate the risk of failure of a system in terms of the probability functions which are applicable to individual components in the system. This

approach provides not only mean values for the probability of failure, but also provides estimates of uncertainties.

Key factors in the calculations include rate of failure data for certain types of valves, especially motor driven valves and manual ball valves. The data for the motor driven valves are derived from Licensee Event Reports. Imperfections in these data make it advisable to calculate lower and upper bounds. For the manual ball valves the most recent data source lists two separate series of rates of failure. This suggests that there may not be a single generic type of manual ball valve. Without additional information, it is prudent to use the series with the greater rate of failure.

Calculation of lower and upper bounds yielded values of probability of the order of 10^{-6} for the lower bound, and 10^{-4} for the upper bound for rates of failure of the hydraulic brake system.

The conclusion reached in the January 1990 report "Probabilities of a Catastrophic Waste Hoist Accident at the Waste Isolation Pilot Plant," EEG-44, by the author remains unchanged.

DOE has erred in the Final Safety Analysis Report in concluding that a catastrophic accident involving the WIPP radioactive waste hoist system over the 25 years of expected operation is incredible (annual probability less than 10^{-6}). DOE should therefore perform consequence analyses of a catastrophic accident involving the waste hoist system. These calculations and mitigation measures to reduce the probability of an accident and to minimize the impact of such an accident should be included in the WIPP Safety Analysis Report.

RECOMMENDATIONS

(1) Since the probability of a catastrophic failure is greater than 10^{-6}, DOE should do a quantitative analysis of such an accident and publish the results in the Final Safety Analysis Report as required by DOE Order AL 5481.1B.

(2) An annual review is advised of all incidents that have occurred which bear on the matters of safety for personnel and quality assurance for equipment and systems. This review should certainly include all Unusual Occurrence Reports (UOR) and Class C Investigations, if any. Based on such a review, consideration should be given to changes that would strengthen quality assurance and minimize human errors in operation of the waste hoist.

(3) The data for failure rates of motor driven valves are based on information developed from 1976 to 1980, some 13 to 17 years ago, the Licensee Event Reports. The authors of the pertinent report, NUREG/CR-2770, point to a number of serious imperfections in the data. Yet it has been shown in the present report that the calculated probability of failure of the waste hoist brake system depends sensitively on the assumed failure rates of motor driven valves. Therefore, it is recommended that the DOE undertake to obtain more current information about the behavior of this crucial type of valve.

(4) A second important type of valve is the manual ball valve. DOE used a reference for failure rates from IEEE Std 500-1984 titled "Nonelectric Parts Reliability Data (NPRD-2)," published in 1981, 12 years ago. The present report cited a later work, NPRD-3, published in 1985. The latter report included the data for manual ball valves which appear in NPRD-2 (used by DOE in their analysis). However, NPRD-3 also included separate data for manual ball valves with failure rates that are 15x those reported in NPRD-2. The question is raised whether there exists more than one type of generic manual ball valve.

Again, it is recommended that the DOE undertake to obtain the most current information about manual ball valves, and to determine specific information about the manual ball valves installed in the brake system of the waste hoist.

(5) If the DOE undertakes additional calculations of the probability of failure of the brake system of the waste hoist, it is recommended that the following items be considered for inclusion:

(a) Follow the recommendations of the Nuclear Regulatory Commission to include mean estimates and to "take into account the potential uncertainties that exist so that an estimate can be made on the confidence level to be ascribed to the quantitative results."

(b) Assume human error can occur. Use the methodology described by Swain and Guttman (1983) which was used in the Chan et al. (1987) report.

I. INTRODUCTION

The hoist for radioactive transuranic waste at the Waste Isolation Pilot Project (WIPP) is a major system, performing important tasks on a daily basis. The Department of Energy (DOE) recognized the importance of evaluating the safety of the waste hoist, and published a number of studies (Banz et al. 1985, Chan et al. 1987, unpublished) which made probabilistic risk assessments of the chance occurrence of a catastrophic accident. Operating experience with the waste hoist led to a number of design changes and improvements. The most recent reports (Westinghouse Electric Corp. 1990, U.S.DOE 1990) summarized an assessment of the risk of an accident at the waste hoist, as it is now constructed. The earlier studies (Banz et al. 1985, Chan et al. 1987) demonstrated that the most important contributor in the accident scenarios was the possible failure of the hydraulic brake system. As a consequence the FSAR and WIPP IRA include an analysis of the likelihood of an accident to the braking system, based on a probabilistic risk assessment.

A major DOE decision (Westinghouse Electric Corp. 1990, p.7B-2) was to describe all results in terms of so-called "point estimates", rather than use probability distributions, which provide estimates of uncertainties. However, within the scientific community, many prefer to use probability distributions, rather than only point estimates, in evaluating risks of failure in a system of interest (Finkel 1990, Apostolakis 1990). This issue will be discussed in some detail later in this report.

There is a second problem with the data used in the DOE report. A major component of the DOE calculations is the information regarding failure rates of certain critical valves in the hydraulic waste hoist system. The DOE report relied on failure rates listed in the IEEE Std 500, 1984. These data came from references describing information gathered from 1976 to 1980. Thus, the failure rate data are largely 10 to 15 years old. In a number of instances these references are superceded by later reports which supply additional information. In one of these references (Steverson and Atwood 1983), there is an important discussion of imperfections in the data, which must be taken into account if one is to have a realistic description of the upper and lower bounds of probabilistic risk assessments.

This report starts with a brief history of the studies completed on the reliability of the hydraulic waste hoist system, followed by discussions of the use of confidence intervals vs. point estimates alone, and the need to consider the possibility of human error as a factor which impacts on the reliability of a system.

This report describes a methodology for calculating probability distributions to evaluate the risk of failure of a system in terms of the probability functions which are applicable to individual components in the system. This approach provides not only mean values for the probability of failure, but also provides estimates of uncertainties.

The report reviews the previous studies made by the DOE, and applies the methodology referred to above to those studies.

II. BRIEF HISTORY OF THE TRANSURANIC WASTE HOIST BRAKE SYSTEM

There have been a number of DOE studies of the waste hoist system. Chan et al. (1987 unpublished draft) placed special emphasis on the brake design, a crucial element from the point of view of safety. Table 1 lists them in chronological order.

(a) The Generic Case Study by Banz et al. (1985) calculated a probability of brake system failure of 3.7×10^{-7}. The EEG reviewed this report and stated that a number of important factors were overlooked, including appropriate quality assurance (QA), quality of maintenance, human factors which may contribute to an accident, and the possibility of operator errors. The DOE rejected these suggestions, especially the idea that human factors may contribute to an accident. Two years later there was a serious incident on July 25, 1987, involving maintenance procedures. Valve No. 45 leaked, and the contractor was called to address the problems, since the hoist was still under warranty.

TABLE 1. STUDIES OF PROBABILITY OF BRAKE SYSTEM FAILURE

Case	Probability of Brake System Failure	Probability of Catastrophic Accident	Source
(a) Generic Case	3.7×10^{-7}	1.7×10^{-8}	Banz et al., 1985
(b) Base Case	2.7×10^{-2}	1.0×10^{-3}	Chan et al., 1987 (unpublished)
(c) Sensitivity Case 1	1.5×10^{-6}	5.2×10^{-8}	Chan et al., 1987 (unpublished)
(d) Design Option			FSAR, App7B 1990
B-2	2.2×10^{-7}		IRA DOE/WIPP -89-010(1990)

The contractor offered to replace the valve with a different type. Some changes allowed the replacement valve to "fit"; after the power was turned on, the hoist freewheeled up 30 ft. (the counterweight is heavier than the hoist conveyance), and then stopped by itself. It was noted that the replacement valve could also be installed in reversed orientation. This was done, the power was turned on, the hoist freewheeled 300 ft., and again it stopped by itself. Fearing the worst, all personnel were ordered to leave the area. The system of braking works as follows: when pressure is "on", the brakes are released; when pressure is "off", the brakes are set. This arrangement was described as being conservative, since it was thought that the probability of having the braking pressure "off" would be greater than the probability of having the braking pressure "on", in case of an accident. The freewheeling incidents appear to undermine this assumption.

The DOE issued an Unusual Occurrence Report (UOR) dated 8/11/87, and a report on a Class C Investigation (Westinghouse Electric Corp., 1987), dated 10/15/87. It appears that when the valve was changed, it was placed in a dead-center position, blocking the possibility of any pressure release. Thus, when the hoist was powered and the brakes released, freewheeling occurred. Sufficient leakage in the hydraulic system could have caused the pressure to drop sufficiently, permitting the brakes to engage.

The UOR listed the mistakes as human errors, which included supervisors, workers, contractor personnel, and absence of proper QA.

(b) The second study by Chan et al. (1987) was an excellent engineering analysis of the defects in the original design, and recommended design changes. They suggested a number of changes in the hydraulic system design, including the addition of two dump valves. If deenergized, these would open and dump fluid directly into reservoirs, relieving pressure in the system, and thus set the brakes. The report analyzed the Generic Case and Base Case, which included the possibility of human errors associated with maintenance procedures. The calculated probability of a brake system failure, called the Base Case, is 2.7×10^{-2}, and the annual probability of a catastrophic accident is 1.0×10^{-3}. Thus, the original claim in the Generic Case of 1.7×10^{-8} (also quoted in the FSAR) was too low by a factor of 6×10^{4}. The very low value of 1.7×10^{-8} in the Generic Case was based on the assumption that two components of the braking system were <u>independent</u>, and that the probability of failure of the braking system, would be the <u>product</u> of the probabilities of failure for each of the two components. In fact the design was such that the two components were <u>not</u> independent in the face of the incident that occurred on July 25, 1987.

(c) Chan et al. (1987) then analyzed the proposed and improved design, called Sensitivity Case 1, and calculated a failure probability value of the brake system as 1.5×10^{-6}. It should be noted that this calculation <u>included</u> the possibility of human errors contributing to the failure of the braking system.

In a report published by the Environmental Evaluation Group (Greenfield 1990), the major criticism of Chan et al. (1987) was their use of point estimates only, with no recognition of data uncertainties. In fact the references used by Chan et al. (1987) included confidence intervals as well as point estimates. However, the confidence intervals were not used in the Chan et al. (1987) report for Sensitivity Case 1.

(d) IRA DOE/WIPP-89-010 (U.S. DOE 1990) describes in detail Design Option B-2, the design adopted by the DOE and built into the waste hoist at this time. This design incorporated the recommendations of the Chan et al. (1987) draft report to add dump valves, and also added additional features to enhance the safety of the brake system and therefore of the waste hoist itself. These new features included the replacement of valves 45 and 51 with types having no "dead center" positions, a crucial hazard in the original system.

The Design Option B-2 calculations resulted in a value for the probability of brake system failure of 2.2×10^{-7}. The details of these calculations are in Table 4-19, page 4-290, Table 4-18, page 4-289, Table 5-3, page 5-8 and Table 5-2, page 5-8, in the IRA (U.S. DOE 1990) report.

The improved hydraulic engineering design, and the careful fault-tree analysis characterizing Design Option B-2 are impressive achievements. However, there are two serious oversights in the analysis of this Option. Once again the report relies only on point estimates, now the mean instead of the median values used previously, without the use of confidence intervals to denote and characterize uncertainties in the data. The second oversight is not considering human error as contributing to the possibility of an accident. This latter point is a regression from the inclusion of human error probabilities (HEP) in the predecessor draft report by Chan et al. (1987). Each of these two omissions will be discussed in some detail.

III. USE OF CONFIDENCE INTERVALS VS. POINT ESTIMATES ALONE

Some key reliability data used in the IRA calculations for Design Option B-2 are from the standard 500-1984 published by the Institute of Electrical and Electronic Engineers (IEEE 1983). This data base usually includes reliability data with median, means and 90% confidence intervals (95% level of confidence).

In the WIPP FSAR (Westinghouse Electric Corp. p. 7B-2), the DOE discusses which calculated quantity should be compared with the DOE standard (the standard is defined by Order AL 5481.1B: "events having a frequency of occurrence of less than one chance in a million per year as being incredible"). They considered as candidates: the median, the mean, and the 95th percentile. With this formulation the DOE restricted their choices to some point estimate, rather than including a confidence interval in addition to a point estimate. The DOE chose to use a mean value, basing their decision in part on a precedent provided by the Nuclear Regulatory Commission. Within its safety goal policy statement, published in the Federal Register (Nuclear Regulatory Commission 1986, p. 30028-30033), the NRC states as its performance guideline that the mean calculated values of individual risk should be compared with the quantitative safety goals. Additionally, for distributions skewed to the right, the mean exceeds the median, making the mean the more conservative choice. However, the DOE made a selective rather than a complete reference on this issue.

A perusal of the referenced Federal Register (Nuclear Regulatory Commission 1986) reveals that indeed a recommendation was made to use mean values. However, the recommendations continue with statements of the need to consider and include, where practicable, estimates of uncertainties! A brief quotation makes the point (see page 30031) (underlining added for emphasis):

> Treatment of Uncertainties
>
> To the extent practicable, the Commission intends to insure that the quantitative techniques used for regulatory decision-making take into account the potential uncertainties that exist so that an estimate can be made on the confidence level to be ascribed to the quantitative results.
>
> The Commission has adopted the use of mean estimates for purposes of implementing the quantitative objectives of this safety goal policy (i.e. the mortality risk objectives). Use of the mean estimates comports with the customary practices for cost-benefit analyses and it is the correct usage for purposes of the mortality risk comparisons. Use of the mean estimates does not however resolve the need to quantify (to the extent reasonable) and understand those important uncertainties involved in the reactor accident risk predictions. A number of uncertainties (e.g. thermal-hydraulic assumptions and the phenomenology of core-melt progression, fission product release and transport, and

containment loads and performance arise because of a direct lack of severe accident experience or knowledge of accident phenomenology along with data related to probability distributions.

In such a situation, it is necessary that proper attention be given not only to the range of uncertainty surrounding probabilistic estimates, but also to the phenomenology that most influences the uncertainties. For this reason, sensitivity studies should be performed to determine those uncertainties most important to the probabilistic estimates. The results of sensitivity studies should be displayed showing, for example, the range of variation together with the underlying science or engineering assumptions that dominate this variation. Depending on the decision needs, the probabilistic results should also be reasonably balanced and supported through use of deterministic arguments. In this way, judgments can be made by the decision maker about the degree of confidence to be given to these estimates and assumptions. This is a key part of the process of determining the degree of regulatory conservatism that may be warranted for particular decisions. This defense-in-depth approach is expected to continue to ensure the protection of public health and safety.

The quotation makes the case clearly and cogently to use a range of uncertainty surrounding all probabilistic estimates. Fortunately, the waste hoist analysis is enormously simpler than that required for analyzing nuclear reactor accidents. For the most part it requires making reasonable assumptions about the performances of a relatively small number of key valves that play a major role in assessing the safety of the brake system. The excellent analysis already made in the IRA identified these valves, and allows one to focus on them. That will be done later in this report.

A concomitant issue, once the need for obtaining confidence intervals is accepted, is the responsibility of the DOE to decide the level of confidence to be used, whether at the 50th percentile, the 95th percentile, the 99th percentile, or even the 99.9th percentile. A forthright statement made by Commissioner Bernthal (Nuclear Regulatory Commission 1986, p. 30033) in the context of the estimated frequency of severe core-damage was: "Conservative consideration of associated uncertainties should offer at least 90 percent confidence (typical good engineering judgment, I would hope) that the offsite release goal is met." This suggests that the DOE should think about how best to meet its safety goals. An expression of a confidence level, plus a justification thereof, would be an important part of the safety goal statement.

IV. HUMAN ERROR PROBABILITY (HEP)

The EEG-44 report describes in some detail the DOE's rejection of the possibility of human error contributing to a waste hoist failure, in the context of the Banz et al. (1985) report. After the freewheeling incidents at the waste hoist on July 25, 1987, the DOE acknowledged the seriousness of human error in two reports: the Unusual Occurrence report, dated 8/11/87, and the Class C Investigation report, dated 10/15/87, analyze the factors, human and others, which contributed to the freewheeling incidents. In fact the next relevant DOE report on the safety of the waste hoist, Chan et al. (1987) does include the possibility of human error in the analyses made. Unfortunately, in the most recent report analyzing the safety of the waste hoist brake system, Design Option B-2, Integrated Risk Assessment (IRA) (U.S. DOE 1990) June 1990, the DOE chose to ignore the possibility of human error, despite occurrences of human error in similar situations.

The section "4.3" of the IRA report, devoted to the "Waste Hoist Brake System Analysis", does discuss the possibility of inclusion of operator action and human error. However, the possibility of including these factors in the accident scenarios is excluded, except under certain stringent conditions. Some of these are discussed now. The following quotes are taken from section "4.3", followed by this writer's comments.

(a) "Proper maintenance is a critical factor in safe system operation. It must be recognized that poor maintenance and maintenance practices have the potential for being a critical contributor to problems. Explicit identification of specific failures, which may occur during equipment maintenance, is beyond the scope of this study" (p.4-13).

<u>EEG Comment:</u> This quote "makes the case" for including poor maintenance practices in accident scenarios. Yet paradoxically the idea is rejected, possibly because of the difficulty in identifying <u>specific</u> failures. A way to

proceed would be to make the reasonable assumption that some instances of poor maintenance practices will occur, and to include an accident scenario with a generic assumption of a poor maintenance practice. The details of how to do this are given in the excellent DOE reference on this subject, used in the Chan et al. (1987) report, "Handbook of Human Reliability Analysis with Emphasis on Nuclear Power Plant Applications", A.D. Swain, H.E. Guttman; NUREG/CR-1278-F, 1983.

(b) "The failure of maintenance personnel to return valves or pump trains to their normal position after valve or pump maintenance will be considered as credible events if: (1) proper positioning can not be detected using required post-maintenance tests, (2) the incorrect position is not immediately detected at the control board by alarm or annunciation, or (3) the component does not receive emergency signal" (p. 4-13).

EEG Comment: One may assume that the required post-maintenance test was not performed, or not performed correctly. Swain and Guttman discuss such possibilities and assign HEP values, including confidence intervals. The DOE reported problems with "annunciation" systems that did not work for reasons associated with human error. The entire point is that the possibilities for human error do exist, reasonable HEP values can be assigned via Swain and Guttman, and inclusion in calculations of cutsets is possible.

(c) "The probability of HE in closing certain vital, well-tagged manual valves that are normally open during operation is assumed to be dominated by their random failure rates. These valves include: 101.1/2, 102.1/2, 103.1/2, 26.1/2, 56.3/6" (p. 4-47).

EEG Comment: The quoted material continues to state that valves 56.3/6 and 26.1/2 are locked in the open position, and only one key is issued at a time to the hoistman. One can imagine human error scenarios here. The hoistman accidentally takes the key home or becomes ill. The extra key may fall into the wrong hand, followed by mischief or sabotage. While the suggested scenario may be far-fetched or unlikely, it is simply not possible to foreclose the possibility of human error due to accident, carelessness, oversight, tiredness, etc. This point will be re-examined when the evidence is considered later in this report about the frequency of valve malfunction instances due to human factors.

(d) "It is assumed that there is no center position for valves 25.1/2/3/4, which block both return flow paths" (p. 4-48).

EEG Comment: Why is an assumption made? It should be known whether these valves do or do not have center blocking positions.

(e) "Poor maintenance practices or miscalibration can have a direct contribution to poor brake system operation. It is assumed that maintenance practices, maintenance procedures, and post-maintenance functional testing is properly performed" (p. 4-50).

EEG Comment: The statement speaks for itself. One assumes and hopes that good practices will occur. However, one must allow for the possibilities of oversight, carelessness, and poor supervision that have occasionally occurred in the past, and may occur in the future. Once again, Swain and Guttman show how to make provision for such possibilities in HEP calculations.

(f) "Human Errors (HEs) associated with failures during maintenance were modeled in previous waste hoist analyses. HEs associated with maintaining valves 51 and 45 and restoration failures were previously modeled. With the system upgrades that have been or will be implemented, the potential for a hoist accident is greatly reduced" (p.4-52).

EEG Comment: Improvements are significant and important. However "greatly reduced" does not equal eliminated! Again the DOE took good care to prevent or mitigate the human errors of the past, particularly those human errors that actually occurred. Does this equate to the elimination of those human errors of the future, that have not occurred in the past?

V. CHAN ET AL. DRAFT REPORT, 1987 (UNPUBLISHED)

The unpublished DOE draft report written by Chan et al. (1987) computes the annual probability of a catastrophic accident at the waste hoist (Sensitivity Case 1; see Table 1). The report bases its calculation on the annual probability that three independent events will occur leading to catastrophic failure. Since the events are independent, the total probability of failure is the product of the probabilities of the independent events. These events and their probabilities are described below:

P_1 = annual probability of electric power failure while hoist is in use.

P_2 = annual probability of failure of either valve 45 or valve 51, due to "local fault" or to operator error during maintenance procedures.

P_3 = annual probability of failure of the dump valves.

P = $P_1 \times P_2 \times P_3$ = annual probability of catastrophic failure of the waste hoist.

Chan et al. used the value $P_1 = 0.034$, taken from an earlier published DOE report, Banz et al. (1985). In an analysis of the report undertaken by the Environmental Evaluation Group, a somewhat different value, $P_1 = 0.0776$, is used for reasons given in the EEG report (#44, 1990). From data in Chan et al. (1987) EEG-44 (Greenfield 1990) obtains the following values for P_2 and P_3:

$$P_2 = 2.7\times10^{-2}$$

$$P_3 = 5.27\times10^{-5}$$

$$\text{Thus, } P = (0.0776)(2.7\times10^{-2})(5.27\times10^{-5}) = 1.10\times10^{-7}$$

This value is somewhat higher than the approximately 5×10^{-8} value for P given by Chan et al. (1987) because of the change in P_1.

The basic criticism of the computation of P in Chan et al. (1987) by EEG-44 is that it is based on point estimates alone, median values in this case, without inclusion of confidence intervals, reflecting uncertainty in the data. It may be helpful to elaborate since this is a key issue.

P_2 failure rates for valves 45 and 51 were based on industry surveys for "local fault" failure rates (IEEE 1983), and on surveys made by Swain and Guttman (1983) for failures due to human errors. In effect by only using the median values in calculating P_2, there exists the possibility that there is a 50% or greater chance that the true value of P_2 is greater than the value chosen. The same situation applies to P_3, the annual failure rate for the dump valves. Again by only using a median value, there is a 50% chance that the true value of P_3 is greater than the value chosen. In fact the calculation of the cumulative distribution function (given later in this report) will show that the value of 1.10×10^{-7} (based on median values) corresponds approximately to the 25th percentile; thus, in this case there is a 75% chance that the true value of P is greater than the value based on a simple use of median values for the components P_2, P_3.

EEG-44 did not attempt a calculation of the cumulative distribution function for P, a point also made by Dr. G. Kaiser (Kaiser 1990). Dr. Kaiser and a colleague, Dr. Gary De Moss, used a Monte Carlo code SAMPLE to make such a calculation. Dr. Kaiser stated that they "used exactly the same data on the probabilities of occurrence of the basic events in the cut sets as were used in EEG-44 together with the same assumptions on the associated uncertainties." The Kaiser calculations were for "Sensitivity Case 1" that appear in the Chan et al. (1987) draft report.

It may be well to make the point that the data and uncertainties that Dr. Kaiser refers to are all taken from Chan et al. (1987) and the references used in that report. The results of the Kaiser calculations are as follows:

TABLE 2. FREQUENCY OF OCCURRENCE OF A CATASTROPHIC WASTE HOIST ACCIDENT (SENSITIVITY CASE 1) (KAISER)

Mean	3.0×10^{-7}
Percentiles	
2.5	2.5×10^{-9}
5.0	2.8×10^{-9}
50.0	1.0×10^{-7}
95.0	1.2×10^{-6}
97.5	1.3×10^{-6}
99.0	4.2×10^{-6}
Chan et al. (1987)	5.2×10^{-8}

Note that Dr. Kaiser's calculation (Table 2) shows a "breach" of the value of 1.0×10^{-6} for the probability of failure at a percentile value of approximately 93. After the calculation of the cumulative distribution function for this case, a comparison will be made with Dr. Kaiser's results; see Table 7.

This report now proceeds to develop the methodology for calculating the cumulative distribution function to describe the probability of a catastrophic waste hoist accident at the WIPP, and applies it to "Sensitivity Case 1" in the Chan et al. (1987) report.

Summary of Problem

Given $P = P_1 \times P_2 \times P_3$ (1)

P is annual probability of catastrophic failure (total system failure).
P_1 is annual probability of electric power failure.
P_2 is annual probability of component failures due to valve malfunctions and human errors.
P_3 is annual probability of failure of dump valves.

P_1 is treated as a constant.
P_2 can be described in terms of sums and products of lognormal distributions.
P_3 can be represented as a function of a lognormal distribution.

P_2 can be expressed as follows:

$$P_2 = P_I + P_{II} + (P_{III\text{-}A})(P_{III\text{-}B}) + (P_{IV\text{-}A})(P_{IV\text{-}B}) \quad (2)$$

P_I = probability of failure of valve 45 due to local faults.
P_{II} = probability of failure of valve 51 due to local faults.
$P_{III\text{-}A}$ = probability of failure of valve 45 due to human error, type A.
$P_{III\text{-}B}$ = probability of failure of valve 45 due to human error, type B.
$P_{IV\text{-}A}$ = probability of failure of valve 51 due to human error, type A.
$P_{IV\text{-}B}$ = probability of failure of valve 51 due to human error, type B.

This formulation for P_2 accounts for 99% of the total brake system failure (Chan et al., 1987, p. iii and Table 4.1-2, p. 37).

All 6 of the functions listed above may be described by lognormal distributions.

A numerical check is now made using the point estimate values (in Chan et al., 1987) employed for the quantities that define P.

TABLE 3. ESTIMATE OF P USING MEDIAN VALUES

	P_1	0.0776 (EEG-44)	
	P_I	9.02×10^{-3}	
	P_{II}	4.52×10^{-3}	
P_{III} =	$P_{III\text{-}A}$	8.1×10^{-2}	Product = 6.56×10^{-3}
$P_{III\text{-}A} \times P_{III\text{-}B}$	$P_{III\text{-}B}$	8.1×10^{-2}	
P_{IV} =	$P_{IV\text{-}A}$	8.1×10^{-2}	Product = 6.56×10^{-3}
$P_{IV\text{-}A} \times P_{IV\text{-}B}$	$P_{IV\text{-}B}$	8.1×10^{-2}	
	$P_{2(equ.2)}$	2.7×10^{-2}	
	P_3	5.27×10^{-5}	
$P = P_1 \times P_2 \times P_3 = 1.1\times10^{-7}$			

The references used by Chan et al. (1987), from which the above data are derived, also state that these numbers (components of P_2 and P_3) are the median values of lognormal distributions; the references also give the 95 percentile values for the distributions.

The following table lists the median and 95 percentile values and the sources.

TABLE 4. DATA SOURCES

Term	Median Value	95th Percentile	Source
P_I	9.02×10^{-3}	4 × Med. Val.	IEEE Std 500-1984, p. 1150
P_{II}	4.52×10^{-3}	4 × Med. Val.	IEEE Std 500 1984, p. 1150
P_{III-A}	8.1×10^{-2}	5 × Med. Val.	Swain and Guttman, 1983
P_{III-B}	8.1×10^{-2}	5 × Med. Val.	Swain and Guttman, 1983
P_{IV-A}	8.1×10^{-2}	5 × Med. Val.	Swain and Guttman, 1983
P_{IV-B}	8.1×10^{-2}	5 × Med. Val.	Swain and Guttman, 1983
P_3	5.27×10^{-5}	4 × Med. Val.	IEEE Std 500-1984, p. 1150

Since the 7 terms in Table 4 correspond to lognormal distributions, it is now possible to calculate all pertinent parameters in terms of the above data.

A general form for the lognormal distribution with the two parameters, μ,σ is given by (Aitchison and Brown 1969):

$$d\Lambda(x) = 1/x\sigma\sqrt{2\pi}\ \exp\{-1/2\sigma^2(\log x - \mu)^2\}dx \qquad (3)$$

Where Λ is the cumulative distribution function.
The median of the distribution is given by: $x = e^{\mu}$
The mean is given by: $x = e^{\mu + 1/2\sigma^2}$

The mode is given by: $x = e\mu - \sigma^2$

The 95 percentile by: $x = e^{\mu + 1.645\sigma}$

i. e., the 95 percentile = (median value) $e^{1.645\sigma}$.

It is clear that from the data in the Table 4, one may calculate the two defining parameters (μ, σ) for each of the 7 lognormal distributions. For convenience in calculating: P_I and P_{II} are multiplied by 10^3; $P_{III\text{-}A}$, $P_{III\text{-}B}$, $P_{IV\text{-}A}$, $P_{IV\text{-}B}$ are multiplied by $10^{3/2}$ (since the A, B terms are multiplied later, the product will have been multiplied by 10^3); P_3 is multiplied by 10^5.

TABLE 5. PARAMETERS IN LOGNORMAL DISTRIBUTIONS

P_i	e^{μ} (median)	μ	σ	σ^2	$e^{\mu+1/2\sigma^2}$ (mean)
I	9.02	2.1995	0.8427	0.7101	12.8652
II	4.52	1.5085	0.8427	0.7101	6.4469
III-A	2.56	0.9405	0.9784	0.9573	4.1336
III-B	2.56	0.9405	0.9784	0.9573	4.1336
IV-A	2.56	0.9405	0.9784	0.9573	4.1336
IV-B	2.56	0.9405	0.9784	0.9573	4.1336
P_3	5.27	1.662	0.8427	0.7101	7.5162

Return to equation (1) and note that it is the product $P_2 \times P_3$ that enters into consideration. Combining with equation (2):

$$\begin{aligned} P_2 \times P_3 &= P_I \times P_3 + P_{II} \times P_3 + (P_{III\text{-}A})(P_{III\text{-}B}) \times P_3 + \\ &\quad (P_{IV\text{-}A})(P_{IV\text{-}B}) \times P_3 \\ &= P_I \times P_3 + P_{II} \times P_3 + P_{III} \times P_3 + P_{IV} \times P_3 \end{aligned} \tag{4}$$

Thus, one has the sum of 4 terms, each consisting of the product of 2 or 3 lognormal distribution functions.

Using a theorem from Aitchison and Brown (Theorem 2.2, p. 11), the product of lognormal distributions is also a lognormal distribution. (This theorem is called the "reproductive theorem").

If X_1 is $\Lambda(\mu_1,\sigma_1^2)$; X_2 is $\Lambda(\mu_2, \sigma_2^2)$; X_3 is $\Lambda(\mu_3,\sigma_3^2)$

Then X_1X_2 is $\Lambda(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ (5)

Also $X_1X_2X_3$ is $\Lambda(\mu_1 + \mu_2 + \mu_3, \sigma_1^2 + \sigma_2^2 + \sigma_3^2)$ (6)

One may now construct a table for the 4 terms in equation 4.

TABLE 6. COMBINED PARAMETERS IN LOGNORMAL DISTRIBUTIONS SENSITIVITY CASE 1

P_i	μ	σ^2	σ	e^{μ} (median)	$e^{\mu+1/2\sigma^2}$ (mean)
$P_I \times P_3$	3.8610	1.4202	1.1917	47.5128	96.64
$P_{II} \times P_3$	3.1705	1.4202	1.1917	23.8194	48.45
$P_{III} \times P_3$	3.5430	2.6246	1.6201	34.5705	128.43
$P_{IV} \times P_3$	3.5430	2.6246	1.6201	34.5705	128.43

Note: The factor 10^{-3} was removed from the terms P_I, P_{II}, P_{III}, P_{IV}, also the factor 10^{-5} was removed from the term P_3. Thus, the factor 10^{-8} is applied to the entire entity in equation (4), for calculations based on Table 6. As a check on the numerical calculations, compute the sum of the median values of the 4 terms in Table 6. It should be equal to the product of the median values for P_2 and P_3.

P_2, median = 2.7×10^{-2}

P_3, median = 5.27×10^{-5}

P_2, median × P_3, median = 1.42×10^{-6}

Compare with $10^{-8} \times$(sum of "median" values in Table 6)=1.40×10^{-6}. Check is satisfactory; the difference is due to rounding off errors.

The problem of describing the distribution function for $P_2 \times P_3$ in equation (4) has now been reduced to determining the distribution function of a variable which is the sum of four random variables, each of which is described by a lognormal function. We compute the distribution function using Fast Fourier Transforms and the calculus of characteristic functions. See Appendix 1.

Figure 1 shows the density function for the sum of the four lognormal distributions. Figure 2 displays the cumulative distribution function for the sum of the four lognormal distributions. As stated previously the factor 10^{-8} must be introduced to obtain $P_2 \times P_3$, the probability of brake system failure. To complete the calculation for P, the probability of catastrophic waste hoist failure, one must also introduce the factor P_1, taken as 0.0776 in order to make a comparison with the calculation of Dr. Kaiser.

Table 7 lists the mode, median, mean values and the various quantiles taken from the computed distribution function for $P_2 \times P_3$. The other two columns list the values for P, and the data from Dr. Kaiser, for comparison.

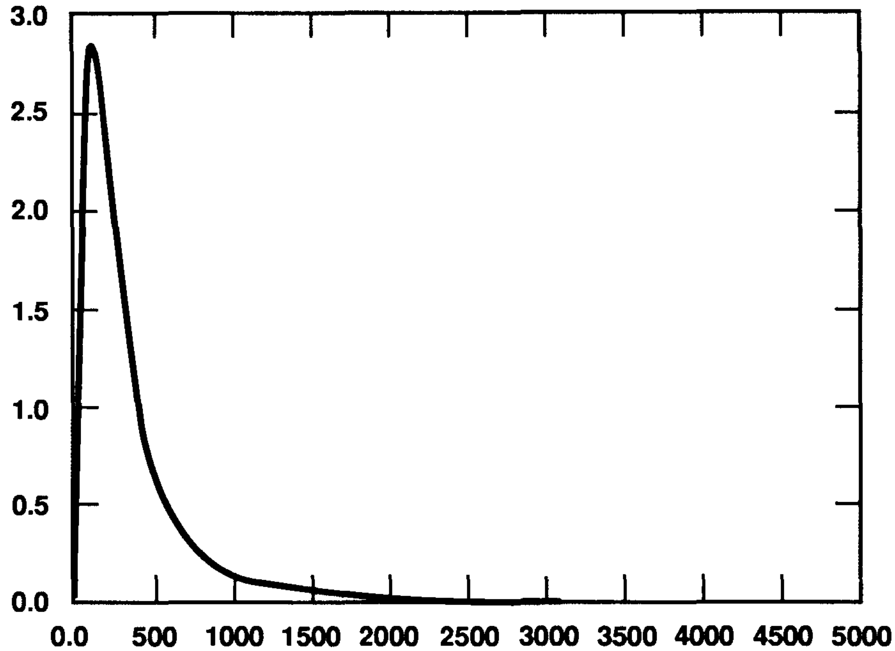

Figure 1. Density function in arbitrary units for the sum of four lognormal distributions. The four log normal distributions have parameters μ_1=3.8610, σ_1=1.1917, μ_2=3.1705, σ_2=1.1917, μ_3=3.5430, σ_3=1.6201, μ_4=3.5430, σ_4=1.6201, respectively.

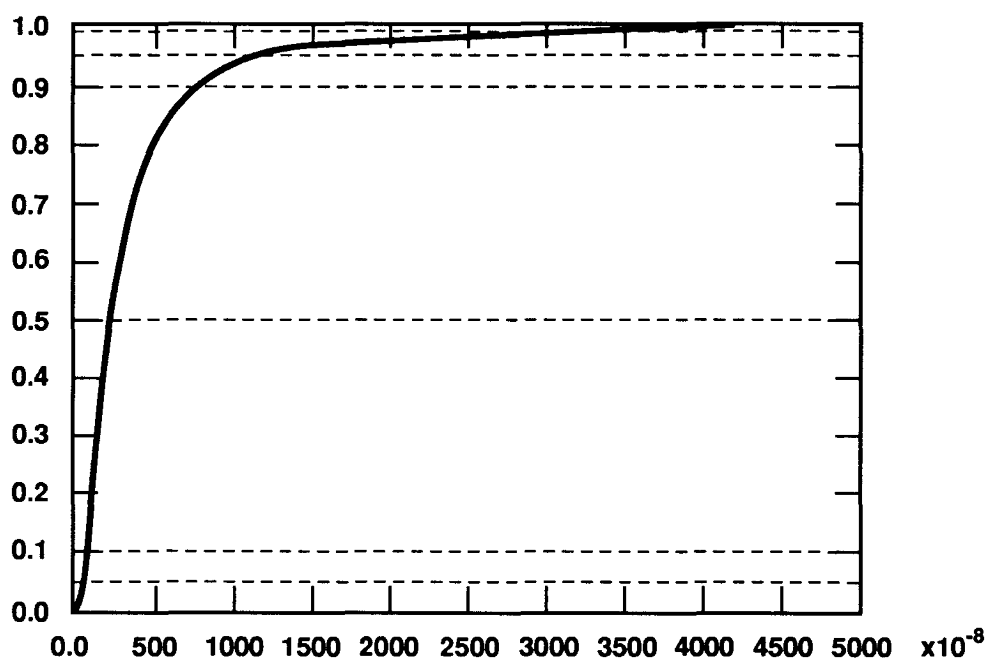

Figure 2. Cumulative distribution function for the sum of four lognormal distributions. The horizontal lines depict quantiles at the 5,10, 50, 90, 95, and 99 percent levels. The factor 10^{-8} is introduced to convert the abscissa values to probabilities.

Density and CDF for probability of failure of brake system for Sensitivity Case 1.

Fig 1&2

TABLE 7. SENSITIVITY CASE 1: COMPARISON OF RESULTS

Percentile	Probability of Failure of Brake System $P_2 \times P_3$	Probability of Catastrophic Waste Hoist Failure $P = P_1P_2P_3$	Probability of Catastrophic Waste Hoist Failure P (Kaiser)
2.0	$50.\times10^{-8}$	3.9×10^{-8}	
2.5	53.5×10^{-8}	4.2×10^{-8}	2.5×10^{-9}
5.0	68.0×10^{-8}	5.3×10^{-8}	2.8×10^{-9}
10	89.5×10^{-8}	6.9×10^{-8}	
Mode 19.9	125.5×10^{-8}	9.7×10^{-8}	
Median 50	248.5×10^{-8}	1.9×10^{-7}	1.0×10^{-7}
Mean 71	401.55×10^{-8}	3.1×10^{-7}	3.0×10^{-7}*
90	794.5×10^{-8}	0.62×10^{-6}	
95	1163.5×10^{-8}	0.9×10^{-6}	1.2×10^{-6}
95.8	1288.7×10^{-8}	1.0×10^{-6}	
97.5	1664.5×10^{-8}	1.3×10^{-6}	1.3×10^{-6}
98	1861.0×10^{-8}	1.4×10^{-6}	
99	2611.5×10^{-8}	2.0×10^{-6}	4.2×10^{-6}
99.9	7405.0×10^{-8}	5.7×10^{-6}	

*Dr. Kaiser did not list a percentile value for the mean.

Figure 3 is a graph of the cumulative distribution function (CDF) for P, the probability of a catastrophic waste hoist accident at the WIPP for Sensitivity Case 1. Note that if one were to use the mean value as representative of the probability for a catastrophic accident, there would be a 29% chance that the true value is greater. Also, note that the value for P of 1.10×10^{-7} based on point estimates alone, corresponds to a percentile of 26%. Thus there is a 74% chance that the true value of P exceeds 1.10×10^{-7}.

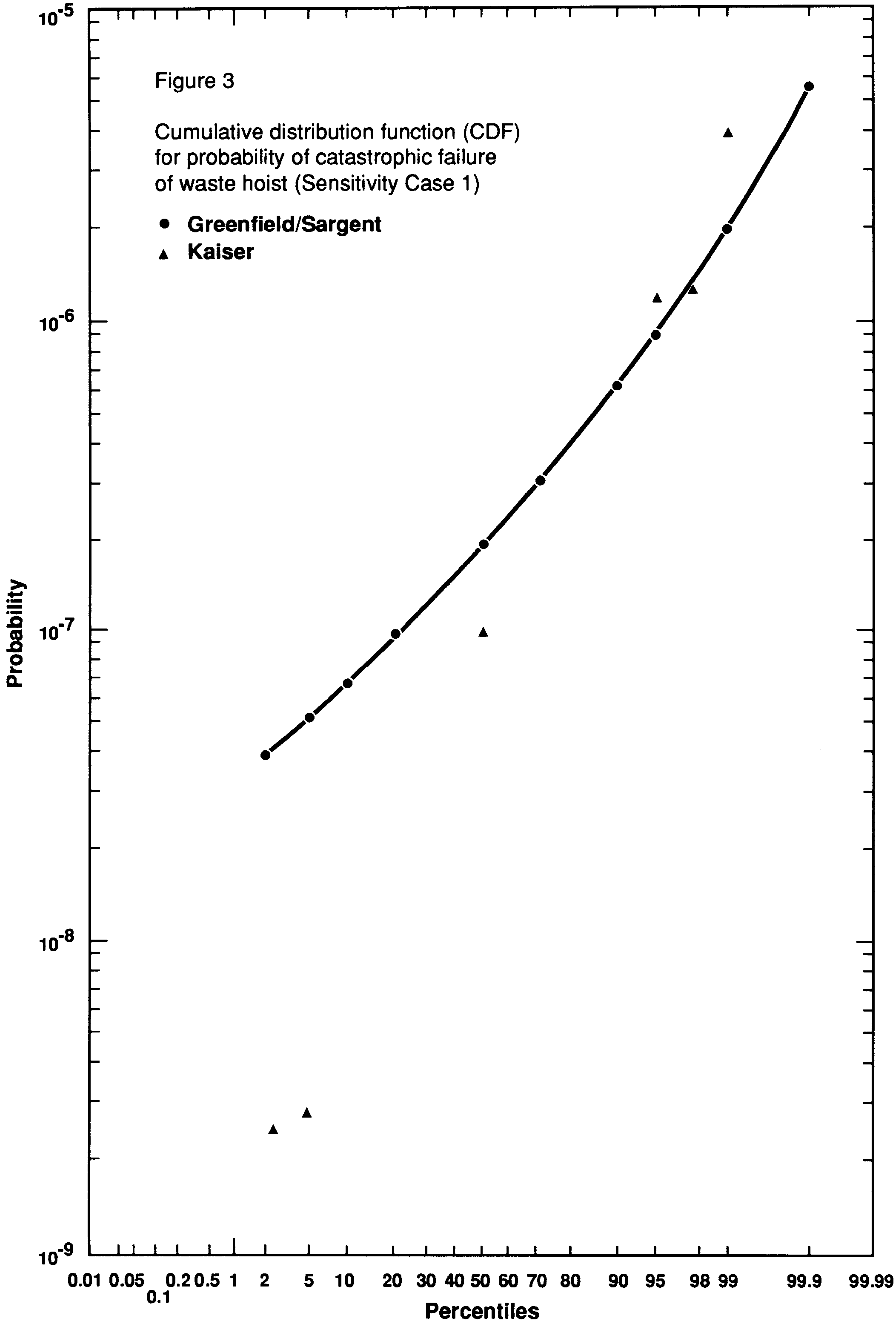
Figure 3
Cumulative distribution function (CDF)
for probability of catastrophic failure
of waste hoist (Sensitivity Case 1)
Greenfield/Sargent
Kaiser
Probability
10-5
10-6
10-7
10-8
10-9
0.01 0.05 0.1 0.2 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.9 99.99
Percentiles

VI. DESIGN OPTION B-2, FSAR, MAY 1990, VOLUME III, CHAPTER 7, APPENDIX 7B

As indicated in Table 1, the FSAR published a Design Option B-2, and had calculated an annual probability of failure for the hydraulic brake system of 2.2×10^{-7}. The details of the DOE calculation are summarized in the IRA (U.S. DOE 1990) in Table 4-19, pages 4-290, 291 under the title "Fault Tree Quantification Results-Option B-2."

The analysis demonstrates the brake system failure rate is dependent upon the failure rates for three main types of valves used in the hydraulic brake system. These are listed in Table 8 along with the identification numbers of the specific valves of each type used in the brake system. Also listed are the data sources for the valve failure rates cited by the DOE, and additional data sources for valve failure rates that were not cited by the DOE.

VALVE FAILURE RATES

Table 8 lists failure rates given by the references in IEEE Std 500-1984. The values for the globe valves have the following significances. The IEEE standard assumes the data can be represented by a log-normal distribution, with the REC (recommended) value equal to the median of the distribution. The high and low values represent the 95 and 5 percentiles respectively, thus defining a 90% interval. The IEEE-sources offered no additional information for the ball and motor operated valves, except for the listed REC values. The IRA DOE/WIPP-89-010, 1990, chose point estimates, calculating a mean value from the data for the globe valves, and simply adopting the REC values for their point estimates for the ball and motor operated valves. With these assumptions DOE arrived at an annual failure rate, given above, of 2.2×10^{-7}.

TABLE 8. DATA FOR VALVES IN DESIGN OPTION B-2

Valve Types	Globe Valves, Solenoid (2 - 3.99 in)	Manual Ball Valves	Motor Operated Valves
Valve Numbers	25.1, 25.2, 25.3, 25.4	56.3, 56.6	108, 45, 51
Data Sources Cited by DOE	IEEE Std 500-1984 p. 1150; ref.15	IEEE Std 500-1984 p. 1044; ref. 4	IEEE Std 500-1984 p. 1023; ref. 2
Failure Rate	High 11.6×10^{-6} 1/hr Rec 2.89×10^{-6} 1/hr Low 0.72×10^{-6} 1/hr	Rec 0.65×10^{-6} 1/hr	Rec 6×10^{-8} 1/hr
IEEE Reference	#15: NUREG/CR-2232, Nuclear Plant Reliability Data System (NPRDS) 1980 Annual Reports of Cumulative System and Component Reliability	#4: Nonelectric Parts Reliability Data (NPRD-2) Summer 1981, Reliability Analysis Center, Rome Air Development Center, Griffis A.F.B., N.Y. 13441-5700	#2: Interim EGG-EA 5B16, April 1982, Data Summaries of Licensee Event Reports of Valves at U.S. Commercial Nuclear Power Plants
Additional Data Sources Not Cited By DOE		Nonelectronic Parts Reliability Data (NPRD-3) 1985 Reliability Analysis Center, Rome Air Development Center Griffis A.F.B., N Y 13441-5700	NUREG/CR-1363, EGG-EA-5816, Rev 1, April 1982, Data Summaries of Licensee Event Reports of Valves at U.S. Commercial Nuclear Power Plants from Jan. 1, 1976 to Dec. 31, 1980, C. F. Miller, W. M. Hubble, M. Trojovsky, S. R. Brown NUREG/CR-2770, EGG EA-5485 RG, Feb. 1983, Common Cause Fault Rates for Valves, Estimates based on Licensee Event Reports at U.S. Commercial Nuclear Power Plants, 1976-1980 J. A. Steverson, C. L. Atwood

A comparison between the improved Design Option B-2 and Sensitivity Case 1 reported in Chan et al. (1987) follows. The data for the Sensitivity Case 1 appeared in Table 7, with the probability for failure of the brake system listed as column 2. It is this probability that is desirable for comparison purposes, since it is the quantity calculated for the Design Option B-2. It will be recalled that in the calculation made for the Sensitivity Case 1, all the component probability functions employed were lognormal. In order to make the comparison with the Design Option B-2 case, it is assumed that all the valve failure rates for Design Option B-2 can be described as lognormal functions, with the listed REC values treated as median values. It will also be assumed that the multiplier, divisor factors to obtain 5 and 95 percentiles, are the same for the ball and motor operated valves as for the globe valves. Later in this report information from other data sources will be utilized in additional computations that are more appropriate for ball and motor operated values.

For clarity in following the computations for Design Option B-2, the same format will be used as in Table 4-19, pages 4-290, 291 of the IRA (U.S. DOE 1990), Volume III, Chapter 7, Appendix 7B (see Table 9). For convenience the Design Option B-2 is simply called Case I.

In Table 9 the first column lists the cutset followed by the valve number and type in columns 2 and 3.* The fourth column lists the hourly failure rate. If the event is due to common cause, then the appropriate value of b is used to obtain P_{cc} in column 5. (Note: in cutset 1(a): b = 0.1; in 2(a): b = 0.04; for 3(a): b = 0.1). An annual service time of 2,048 hours is assumed, and this is multiplied by Q or P_{cc} to obtain the Event Probability per annum (EP) as column 6.

As a convenience to obtain numbers close to unity, introduce the multiplier 10^9 to obtain column 7. Since each set of the cutset components (a),(b) or (a),(b),(c) are ultimately to be multiplied, the multiplier 10^9 is distributed

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*The cutset number represents a mode of possible failure due to the failures of the valve types listed. Thus the cutset numbers enumerate the possible failure modes which are deduced from a fault tree analysis.

among the components in column 6 to obtain column 7. Recall that for log-normal distributions the median value is given by e^{μ} where μ is one of the lognormal parameters. Thus column 7 is called e^{μ}, and column 8 yields μ.

TABLE 9. CALCULATIONS FOR LOGNORMAL PARAMETERS, DESIGN OPTION B-2 (CASE I)

Cutset No.	Valve No.	Valve Type	Q median 1/hr	$P_{cc} = bQ$ 1/hr	E.P.	10^9×E.P $= e^{\mu}$	μ	σ
1 (a)	25.2.4	Globe; Motor	2.89×10^{-6}	2.89×10^{-7}	5.9×10^{-4}	5.9	1.77	0.8448
(b)	108	Operated	6.00×10^{-8}		1.23×10^{-4}	12.3	2.51	0.8448
2 (a)	25.1.2 25.3.4	Globe;	2.89×10^{-6}	0.1156×10^{-6}	2.4×10^{-4}	2.4	0.88	0.8448
(b)	108	Motor Operated	6.00×10^{-8}		1.23×10^{-4}	12.3	2.51	0.8448
3 (a)	56.3.6	Ball; Motor	6.50×10^{-7}	6.50×10^{-8}	1.33×10^{-4}	1.33	0.285	0.8448
(b)	108	Operated	6.00×10^{-8}		1.23×10^{-4}	12.3	2.51	0.8448
4 (a)	51	Motor Operated;	6.00×10^{-8}		1.23×10^{-4}	1.23	0.207	0.8448
(b)	108	Motor Operated	6.00×10^{-8}		1.23×10^{-4}	12.3	2.51	0.8448
5 (a)	45	Motor Operated;	6.00×10^{-8}		1.23×10^{-4}	1.23	0.207	0.8448
(b)	108	Motor Operated	6.00×10^{-8}		1.23×10^{-4}	12.3	2.51	0.8448
6 (a)	25.4	Globe; Motor	2.89×10^{-6}		5.9×10^{-3}	5.9	1.77	0.8448
(b)	108	Operated;	6.00×10^{-8}		1.23×10^{-4}	0.123	-2.096	0.8448
(c)	25.2	Globe	2.89×10^{-6}		5.9×10^{-3}	5.9	1.77	0.8448
7 (a)	25.4	Globe; Motor	2.89×10^{-6}		5.9×10^{-3}	5.9	1.77	0.8448
(b)	108	Operated;	6.00×10^{-8}		1.23×10^{-4}	1.23	0.207	0.8448
(c)	56.3	Ball	6.50×10^{-7}		1.33×10^{-3}	0.133	-2.017	0.8448

Cutsets 8 through 14 are numerically the same as cutset 7.

Cutsets 1-14 account for 99% of the total of 26 cutsets in Table 4-19, and are considered a sufficiently good approximation to the total of 26 cutsets in FSAR Table 4-19.

The value of σ may be obtained by using the relations between the median value, and the 5,95 percentiles:

$$UB/median = e^{1.645\sigma}$$

$$\begin{aligned} \text{Thus } 11.6/2.89 = 4.0138 &= e^{1.645\sigma} \\ 1.645\sigma &= 1.3897 \\ \sigma &= 0.8448 \end{aligned}$$

This value of σ is listed in the last column. [Note: In the calculation for the Sensitivity Case 1 (Chan et al. 1987), the ratio of the 95 percentile to the median was rounded off to 4 (Table 4). This led to the slightly smaller value of $\sigma = 0.8427$ used in Table 5.]

A numerical comparison with the value of 2.12×10^{-7} obtained in Table 4-19 of the IRA (U.S. DOE 1990) can be made by summing the EP values in Table 9. However to be consistent with Table 4-19 one must use mean values instead of the medians for the globe terms. These terms appear in cutsets 1(a), 2(a), 6(a), 6(c), 7(a). The mean values are computed as follows:

$$\text{Mean} = \text{Median} \times e^{1/2\sigma 2}$$

$$\text{with } \sigma = 0.8448$$

$$e^{1/2\sigma 2} = 1.4288$$

$$\text{Thus } 5.9 \times 1.4288 = 8.4$$

$$2.4 \times 1.4288 = 3.4$$

$$\begin{aligned} \text{Then P} &= 10^{-9}\, [8.4 \times 12.3 + 3.4 \times 12.3 + 1.33 \times 12.3 \\ &\quad + 1.23 \times 12.3 + 1.23 \times 12.3 + 8.4 \times 0.123 \times 8.4 \\ &\quad + 8 \times 8.4 \times 1.23 \times 0.133] \\ &= 211\times10^{-9} = 2.11\times10^{-7} \end{aligned}$$

This is in good agreement with the value of 2.12×10^{-7} of Table 4-19 of the FSAR.

Table 10 lists the values of the μ, σ parameters derived in Table 9.

TABLE 10. VALUES OF THE PARAMETERS μ, σ FOR DESIGN OPTION B-2 (CASE I)

P_i	μ	σ	P_i	μ	σ
I-a -b	1.77 2.51	0.8448 0.8448	I	4.28	1.1947
II-a -b	0.88 2.51	0.8448 0.8448	II	3.39	1.1947
III-a -b	0.285 2.51	0.8448 0.8448	III	2.795	1.1947
IV-a -b	0.207 2.51	0.8448 0.8448	IV	2.717	1.1947
V-a -b	0.207 2.51	0.8448 0.8448	V	2.717	1.1947
VI-a -b -c	1.77 -2.096 1.77	0.8448 0.8448 0.8448	VI	1.444	1.4632
VII-a -b -c	1.77 0.207 -2.017	0.8448 0.8448 0.8448	VII	-0.040	1.4632

Note: VIII, IX, X, XI, XII, XIII, XIV are the same as VII. Since P_{i-a} is multiplied by P_{i-b} etc., one can use the "reproductive theorem." See equations (5) and (6) to obtain the right hand part of Table 10.

$$\text{Thus} \quad P = 10^{-9} \times \sum_{i=I}^{i=XIV} P_i$$

The problem is now reduced to computing the density function and cumulative distribution function of a random variate, P, which is defined as the sum of fourteen independent random variables, each of which is log normally distributed, with parameters μ, σ listed in Table 10. See Appendix 1 for the methodology of the calculation. The probability density function for the Design Option B-2 case (Case I) is given in Figure 4. The cumulative distribution function (CDF) is given in Figure 5. The factor 10^{-9} is introduced to convert the abscissa values to probabilities. Figure 6 gives the cumulative distribution function for probability of failure of the brake system.

TABLE 11. PROBABILITY OF FAILURE OF BRAKE SYSTEM VS. PERCENTILES FOR DESIGN OPTION B-2 (CASE I), SENSITIVITY CASE 1

Percentile	Probability of Failure (Case I)	Probability of Failure Sensitivity Case 1
1.0	0.077×10^{-6}	
10.0	0.130×10^{-6}	0.90×10^{-6}
50.0	0.263×10^{-6}	2.48×10^{-6}
Mean 66.5	0.341×10^{-6}	4.01×10^{-6} 71 percentile
85.0	0.513×10^{-6}	
95.0	0.811×10^{-6}	11.6×10^{-6}
99.0	1.48×10^{-6}	26.1×10^{-6}
99.78	2.49×10^{-6}	
99.9		74.0×10^{-6}

Table 11 lists the failure probabilities for both Case I and Sensitivity Case 1. Note that the mean for Case I is less than that for the Sensitivity Case 1 by almost a factor of 12. The IRA (U.S. DOE 1990) value of the probability of failure for the brake system for Case I is 2.2×10^{-7}. Referring to this value of probability in Figure 6 for Case I, the percentile is 36%. Thus there is a 64% likelihood that the failure rate is greater than a rate based on point estimates alone.

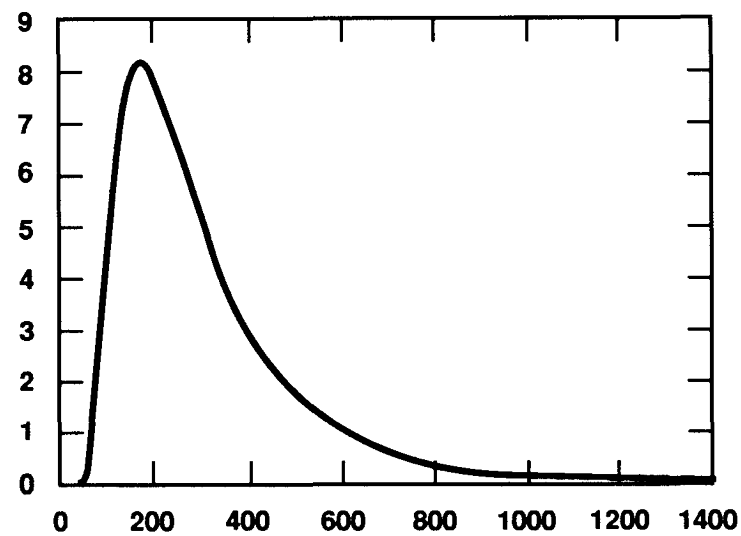

Figure 4. Density function in arbitrary units for Case I.

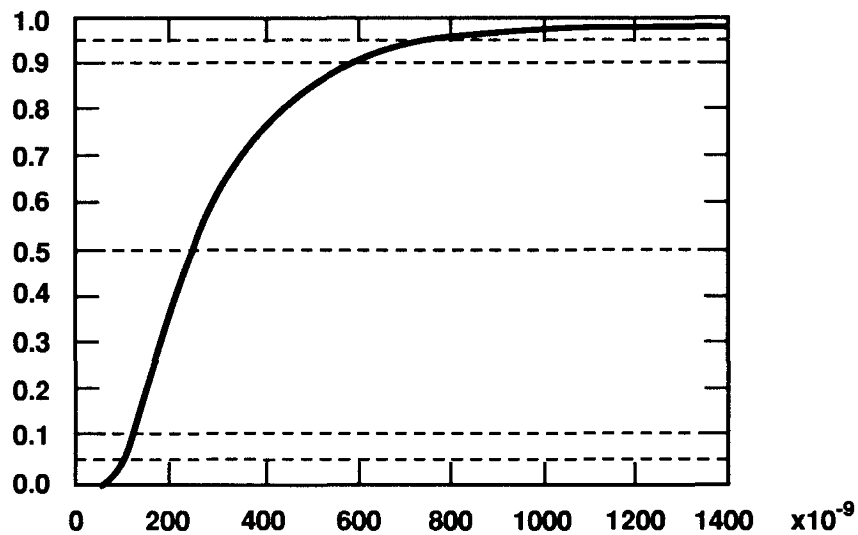

Figure 5. Cumulative distribution function for Case I. The factor 10^{-9} is introduced to convert abscissa values to probabilities.

Density and CDF for probability of failure of brake system, Case I (Design Option B-2).

Fig 4&5

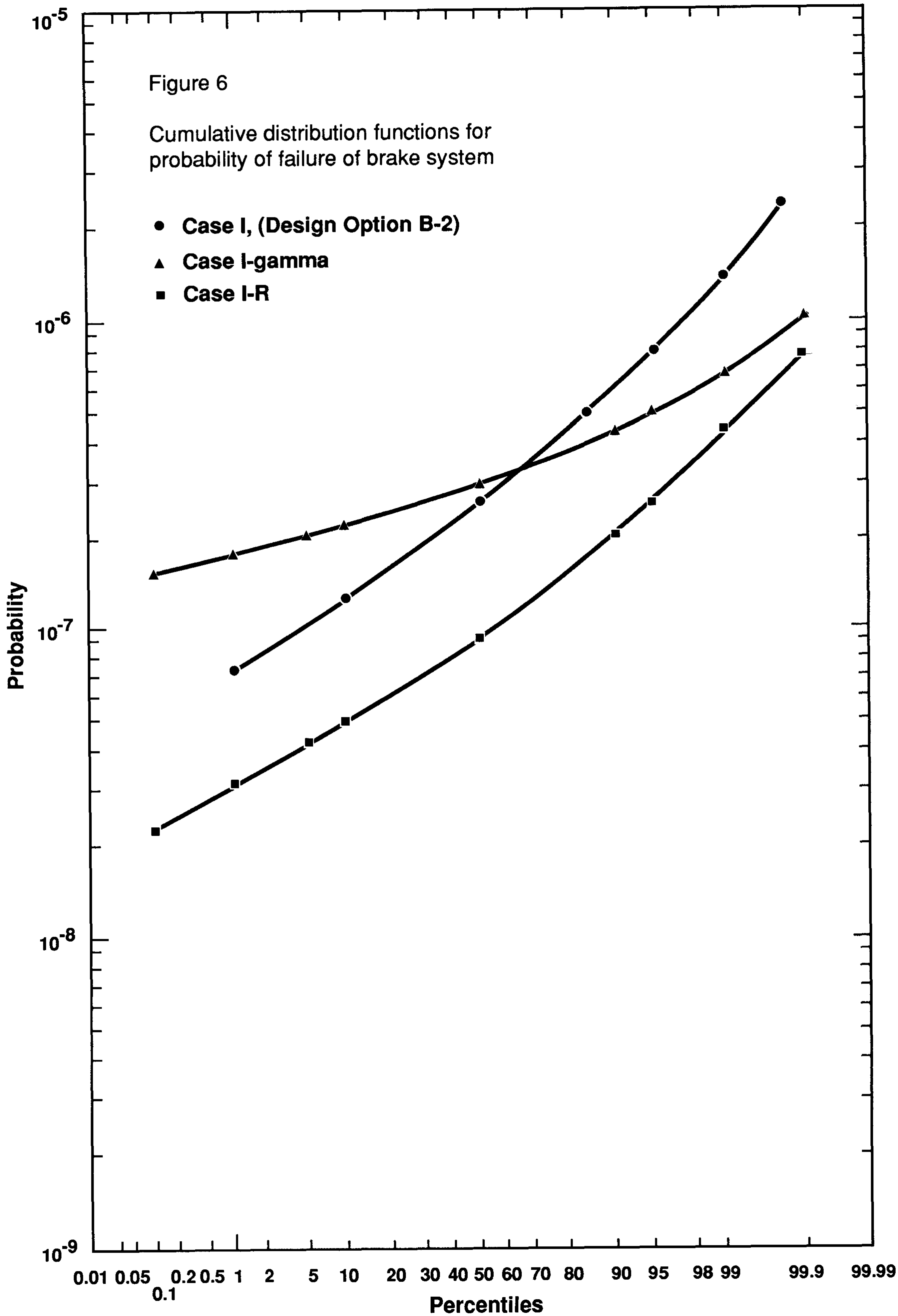

Figure 6
Cumulative distribution functions for
probability of failure of brake system
Case I, (Design Option B-2)
Case I-gamma
Case I-R
Probability
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0.01 0.05 0.1 0.2 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.9 99.99
Percentiles

VII. DATA SUMMARIES AND UNCERTAINTIES OF LICENSEE EVENT REPORTS OF VALVES AT U.S. COMMERCIAL NUCLEAR POWER PLANTS: VALUES OF PARAMETERS FOR MOTOR OPERATED VALVES

Table 9 shows the importance of valve 108, a motor operated type, since it appears in all the cut sets. In fact in the FSAR, Table 4-20, page 4-292, valve 108 is listed as the most important component in the analysis of failure modes for option B-2. Thus, all information regarding the performance and failure rates of motor operated valves is important to this study.

The only reference for this type of valve in the DOE study was to the IEEE Std 500-1984, p. 1023, reference 2, (see Table 8). As indicated in Table 8 that reference was to an interim report published by EG&G as EGG-EA-5B16, April 1982, Data Summaries of Licensee Event Reports of Valves at U.S. Commercial Nuclear Power Plants. The data from page 1023 is included for convenience as Table 12. A revision with more data was also published by EG&G as NUREG/CR-1363, EGG-EA-5816, Rev. 1, April 1982 by C. F. Miller, W. H. Hubble, M. Trojovsky, and S. R. Brown. The title is almost the same as the interim report, but with some additional specifics of the time frame covered, "Data Summaries of Licensee Event Reports of Valves at U.S. Commercial Nuclear Power Plants; January 1, 1976 to December 31, 1980."

Mr. Miller granted permission for using page 386 from NUREG/CR-1363, see Table 13. Note that Table 13 is also for motor operated valves (plugged), the same as Table 12. Also, Table 13 indicates that command faults are included. Table 13 includes the single datum of 6.0×10^{-8} in Table 12, but has considerably more information. Table 13 indicates that the 6.0×10^{-8} is an estimated upper 95 percent confidence bound with no failures recorded. A number of additional data points are included in Table 13, along with 5% and 95% confidence factors. Possibly the best representative set for this table is the "overall category", which includes an LER rate estimate along with 5% and 95% confidence factors. Since the data from NUREG/CR-1363 (Miller et al. 1982) is more inclusive than the IEEE source, a recalculation of the Case I, Design Option B-2 will be made later in this report, using the data from Table 13, "the overall" set of numbers.

TABLE 12. DATA ON FAILURE RATES FOR MOTOR OPERATED VALVES

Source: IEEE Std 500-1984, p 1023	
Failure Mode: Plugged	
Failure/10^6	0.06

TABLE 13. DATA FROM NUREG/CR-1363: EGG-EA-5816, REV. 1 APRIL 1982 (page 386) BY C. F. MILLER, W. H. HUBBLE, M. TROJOVSKY, S. R. BROWN

Final Statistics
Valve--Operator (Motor)--Plugged (Command Faults Included)

	Standby Failure Rate (Failures/Hours)
BAB.&WIL.	2.5×10^{-7}*
COMB.ENG.	4.7 1.1×10^{-7} 19.5
WESTINGH.	2.6 5.7×10^{-8} 3.7
PWR'S	2.3 5.4×10^{-8} 2.9
GE (BWR'S)	6.0×10^{-8}*
OVERALL	(×)2.3 (7.36×10^{-8}) 3.2×10^{-8} (÷)2.9 (1.10×10^{-8})

*denotes upper 95 percent confidence bound when no failures recorded

X,X - Upper 95% Confidence Multiplier
Y,YE-YY - LER Rate Estimate
Z,Z - Lower 5% Confidence Divisor

As indicated in Table 8 there is an additional report from the EG&G group published shortly after NUREG/CR-1363 (Miller et al. 1982). It is NUREG/CR-2770, EGG-EA-5485 RG, February 1983 (Steverson and Atwood 1983). The report title is "Common Cause Fault Rates for Valves, Estimates Based on Licensee Event Reports at U.S. Commercial Nuclear Power Plants, 1976-1980," by authors J. A. Steverson and C. L. Atwood.

As the title suggests, and the authors confirm in their report, NUREG/CR-2770 (Steverson and Atwood 1983) analyzes the data collected and reported in NUREG/CR-1363 (Miller et al. 1982). Indeed the latter report is the first main reference for NUREG/CR-2770 (Steverson and Atwood 1983). Additionally, the authors of NUREG/CR-2770 consulted closely with C. F. Miller, the senior author of NUREG/CR-1363.

An important part of NUREG/CR-2770 is a discussion of imperfections in the data, obtained from Licensee Event Reports (LERs).

> "LER reporting policies may vary from plant to plant."
>
> "An important deficiency in the data is the imperfect population counts . . . ."
>
> "A final major imperfection in the data is lack of precise detail in the LERs. The reports are often so vaguely worded that the <u>type of valve</u>, number of failed valves, failure mode, or failure cause is uncertain" (underlining added for emphasis).

Another important defect in the LERs is the fact that many of them do not specify the valve type, "but do make it clear that the valve is operated remotely. Since motor-operated valves are the most common type, Miller et al., believe that most of the failures of remotely operated valves <u>are actually failures of motor-operated valves</u>." For that reason in NUREG/CR-2770 (Steverson and Atwood 1983) "LERs for remotely operated valves are pooled with those for motor-operated valves, and estimates are given on the basis of the pooled data." The authors then conclude "These estimates are <u>upper bounds</u> on what would be found for motor-operated valves if <u>more complete information</u> were available" (underlining added for emphasis).

In the light of these views, one must regard all calculations as establishing rather rough bounds. The data are not precise enough to permit stronger statements. Clearly the distributions being calculated will reflect the great variability in the data available.

The authors of NUREG/CR-2770 (Steverson and Atwood 1983) found it convenient to use a two-parameter gamma distribution to describe the data, although they state that a lognormal distribution might work equally well. Their data are presented with a mean value and a 90% confidence interval (5% and 95% percentiles).

Data on remote/motor operated valves for failure to remain open (plugged) are contained in the report NUREG/CR-2770 (Steverson and Atwood 1983), and are based on Licensee Event Reports. Some definitions used in the report are in order.

The authors define a failure as "an event in which the valve itself needs repair in order to perform as designed. A command fault is an event in which the valve does not fail, but it does not function as desired due to external inputs or lack of inputs"; see pages 2 and 3 of NUREG/CR-2770 (Steverson and Atwood 1983).

The authors use the term "shock" to denote an event, external to the valve or valves, that can cause failures. They distinguish between non-lethal shocks and lethal shocks. Non-lethal shocks refer to external events which may cause valves in the affected system to fail independently of each other, each with some probability. A lethal shock is defined as an external event which causes every valve in the affected system to fail. However, no such events (lethal shocks) were recorded in the LERs.

Some additional definitions:

Lambda(+) = Rate of non-lethal shocks that cause at least one valve to be inoperable.

R_1 = Rate at which a specific valve becomes inoperable, either as an individual fault or due to a shock.

The data of interest on page 92 of the report, NUREG/CR-2770 (Steverson and Atwood 1983), (see Table 14), include both failures and command faults, consistent with Table 13 from NUREG/CR-1363. The quantity M is the number of components subject to failure in the various systems included in the total data bank. The values listed include the triplet: 5 percentile, mean, 95 percentile.

TABLE 14. COMPARISON OF FAILURE DATA FOR REMOTE/MOTOR OPERATED VALVES

Source	5% Confidence	Mean	95% Confidence
IEEE, p 1023; EGG-EA-5B16 April 1982			6×10^{-8}
NUREG/CR-1363 EGG-EA-5816 Rev 1, April 1982; "overall" Table 13	1.10×10^{-8}	3.2×10^{-8}	7.36×10^{-8}
NUREG/CR-2770 EGG-EA-5485RG Feb. 1983	lambda(+) 0.82×10^{-7}	3.6×10^{-7}	7.9×10^{-7}
	R_1 (M-1 case) 1.5×10^{-7}	4.8×10^{-7}	14×10^{-7}

All the data in Table 14 derive from a single source, the LERs. The data from NUREG/CR-2770 reflect the consequence of pooling the data from remotely operated valves with the motor operated valves, as per the quotation from NUREG/CR-2770 (Steverson and Atwood 1983).

VIII. CALCULATIONS OF CUMULATIVE DISTRIBUTION FUNCTIONS (CDF) BASED ON LICENSEE EVENT REPORTS OF VALVES AT U.S. COMMERCIAL POWER PLANTS

In Chapter VI calculations were made of the CDFs for Case I, utilizing assumed lognormal distributions for the failure data for the manual ball valves and for the motor-operated valves. An arbitrary assumption was also made for the value of σ, a parameter appearing in lognormal distributions. Now more appropriate calculations will be made using failure data from NUREG/CR-1363 (Miller et al. 1982) and from NUREG/CR-2770 (Steverson and Atwood 1983) for the motor-operated valves, and failure data from NPRD-3 (Rossi 1985) for the manual ball valves (Tables 14 and 15). However, a step-by-step process will be followed in introducing the new data, and the new distributions that are appropriate for them. Initially the calculations will be made using the new data for the motor-operated valves, with two parameter gamma distributions, while retaining the old data and lognormal distributions for the ball valves. This will permit an assessment of the impact of the changes associated only with the motor-operated valves. Following those calculations will be additional ones that introduce the NPRD-3 data for the manual ball valves, with the use of gamma and exponential distributions.

The basic failure rates for the motor-operated valves are summarized in Table 14. They include the "overall" rates from NUREG/CR-1363 (Miller et al. 1982). The additional data from NUREG/CR-2770 (Steverson and Atwood 1983) for lambda(+) and R_1 reflect failure rates which include the influence of external factors (shocks), and include the fact that "LERs for remotely operated valves are pooled with those for motor-operated valves, and estimates are given on the basis of the pooled data. These estimates are upper bounds on what would be found for motor-operated valves if more complete information were available" (Steverson and Atwood 1983, p. 6).

METHODOLOGY FOR USING TWO PARAMETER GAMMA DISTRIBUTIONS

The following notation is found in Karl Pearson's "Tables of the Incomplete Gamma Function," Cambridge University Press, 1957.

The frequency curve for which the incomplete gamma function forms the probability integral may be written as:

$$f(x) = \frac{\lambda(\lambda x)^{p}e^{-\lambda x}}{\Gamma\ (p+1)} \quad ; \quad \begin{matrix}\lambda>0 \\ p+1>0\end{matrix} \tag{7}$$

λ and p are parameters which can be related to the mean, $\bar{x}$, and the standard deviation, σ.

It can be shown that the standard deviation, σ, and the mean, $\bar{x}$, are given by:

$$\bar{x} = \frac{p+1}{\lambda} \quad ; \quad \sigma = \frac{\sqrt{p+1}}{\lambda} \tag{8}$$

The probability integral may be written as:

$$I(x) = \frac{\int_0^x \lambda(\lambda x)^{p}e^{-\lambda x}dx}{\Gamma(p+1)}$$

$$\text{Let } v = \lambda x$$

$$I(v) = \frac{\int_0^v v^{p}e^{-v}dv}{\Gamma(p+1)}$$

Pearson then replaces the upper limit value, v, with another parameter, u, defined as follows:

$$v = u\sqrt{p+1}$$

The reason for this substitution may be understood from the following:

$$\text{Since } \sigma = \frac{\sqrt{p+1}}{\lambda} \text{ and } v = \lambda x$$

$$\text{Then } u = \frac{v}{\sqrt{p+1}} = \frac{\lambda x}{\sqrt{p+1}} = \frac{x}{\sigma} \tag{9}$$

Thus u is x expressed in units of the standard deviation, σ. The probability integral may now be written as:

$$I(u,\ p) = \frac{\int_0^{u\sqrt{p+1}} e^{-v} v^p dv}{\Gamma(p+1)} \tag{10}$$

Pearson's tables list the values of I in terms of the two parameters u and p. An example will illustrate how the tables may be used.

Assume that failure data produce the values for the 5 and 95 percentiles as:

$$x_5 = 0.3072 \ ; \ x_{95} = 2.867$$

Note that $x = \sigma u$; then

$$u_{95}/u_5 = x_{95}/x_5 = 2.867/0.3072 = 9.33$$

Note also the values of the Integral I in this case are $I_5 = 0.05$ and $I_{95} = 0.95$. In the tables one finds two values of tabulated p for which the ratio u_{95}/u_5 will bracket the ratio 9.33; e.g., for p = 1.5 and the I values of 0.05 and 0.95, $u_5 = 0.359$ and $u_{95} = 3.50$; the ratio is 9.75. Similarly for p = 1.6 one finds the ratio of $u_{95}/u_5 = 9.25$. By interpolation one may compute p = 1.562. Again, by interpolation with this value of p one may find $u_5 = 0.377$ and $u_{95} = 3.515$. The ratio is a satisfactory 9.32.

One may calculate λ from equation 9:

$$\lambda = \frac{u_5\sqrt{p+1}}{x_5} = 1.963$$

$$\bar{x} = \frac{p+1}{\lambda} = 1.304 \text{ (from equation 8)}$$

As a check a numerical calculation was made of the mean value of the frequency curve, using the above values for p and λ. The result was a satisfactory 1.305.

With the values of λ and p determined, one has the complete description of this particular frequency distribution and of the associated probability integral.

CALCULATIONS FOR CASE I-GAMMA

Calculations of the frequency distribution and CDF for Case I involved using the failure data from the IEEE sources only, plus some assumptions of lognormal distributions for the motor-operated and manual ball valves. The present calculation, Case I-gamma, will make two changes from Case I. A two parameter gamma distribution will be used for the motor-operated valves, and its failure data will be the "overall" data from EGG/CR-1363; see Table 14. Since the mean failure rate of 3.2×10^{-8}(1/hr) is less than the 6.0×10^{-8}(1/hr) rate used in Case I, one expects the probabilities to be less on that account. However the change from a lognormal distribution to a two parameter gamma distribution will have its effect, and the result will be a consequence of both changes.

From Table 14:

Failure rates per hour:

5 percentile:	1.10×10^{-8}
mean:	3.2×10^{-8}
95 percentile:	7.36×10^{-8}

Any two of the three parameters will serve to determine p and λ of the gamma distribution. It was suggested that the mean and 95 percentile values may be more reliable.** This choice alters the detail of the calculation for λ and p slightly from what was discussed previously in this report. (Later in this report both methods are used with similar results.)

**Private communication, Dr. C.L. Atwood

A work load of 2,048 hours per annum is assumed. Thus, the annual rates become:

$$\text{mean:} \quad 3.2\times10^{-8}\times2{,}048 = 0.6554\times10^{-4}$$

$$\text{95 perc:} \quad 7.36\times10^{-8}\times2{,}048 = 1.5073\times10^{-4}$$

Since it is convenient to work with numbers near unity, introducing a multiplier factor of 10^4 permits these definitions of the x quantities:

$$\bar{x} = 10^4\times0.6554\times10^{-4} = 0.6554$$

$$x_{95} = 10^4\times1.5073\times10^{-4} = 1.5073$$

From equations (8) and (9)

$$\bar{x} = \frac{p+1}{\lambda} \qquad (8)$$

$$x = \frac{u\sqrt{p+1}}{\lambda} \qquad (9)$$

In equation (9) let x, u, refer to the 95 percentile:

$$x_{95} = \frac{u_{95}\sqrt{p+1}}{\lambda} \qquad (10)$$

Take the ratio of each side of equations (10) and (8).

$$\frac{x_{95}}{\bar{x}} = \frac{u_{95}}{\sqrt{p+1}} \qquad (11)$$

Since the ratio of the ($x_{95}/\bar{x}$) is known, one may find p from the Pearson Tables. In this case from equation (11):

$$\frac{u_{95}}{\sqrt{p+1}} = \frac{1.5073}{0.6554} = 2.2998$$

From the Tables for $p = 1.2$, $u_{95} = 3.416$

$$\frac{u_{95}}{\sqrt{p+1}} = \frac{3.416}{\sqrt{2.2}} = 2.303$$

This is a sufficiently close approximation. One may now calculate λ from equation (8).

$$\lambda = \frac{p+1}{\bar{x}} = \frac{2.2}{0.6554} = 3.3567$$

Since the system includes valves described by lognormal parameters and by gamma distribution parameters, Table 15 outlines the various parameter values. The values for the globe and ball valves are the same as the EP values listed in Table 9 for Case I.

TABLE 15. CALCULATIONS FOR CASE I-GAMMA

Cutset No.	Valve No.	Valve Type	EP	10^8 EP	μ	σ	λ	p+1
1(a)	25.2.4	globe;	5.8786×10^{-4}	5.8786	1.7713	0.8448		
(b)	108	motor	0.6554×10^{-4}	0.6554				
		operated	1.5073×10^{-4}	1.5073			3.3567	2.20
2(a)	25.1.2	globe;	2.4354×10^{-4}	2.4354	0.8901	0.8448		
	25.3.4							
(b)	108	motor		0.6554				
		operated		1.5073			3.3567	2.20
3(a)	56.3.6	ball;	1.33×10^{-4}	1.33	0.2852	0.8448		
(b)	108	motor		0.6554				
		operated		1.5073			3.3567	2.20
4(a)	51	motor		0.6554				
		operated;		1.5073			3.3567	2.20
(b)	108	motor		0.6554				
		operated		1.5073			3.3567	2.20
5(a)	45	motor		0.6554				
		operated;		1.5073			3.3567	2.20
(b)	108	motor		0.6554				
		operated		1.5073			3.3567	2.20
6(a)	25.4	globe;	5.8786×10^{-3}	5.8786	1.7713	0.8448		
(b)	108	motor		0.6554				
		operated;		1.5073			3.3567	2.20
(c)	25.2	globe	5.8786×10^{-3}	5.8786×10^{-2}	-2.8339	0.8448		
7(a)	25.4	globe;	5.8786×10^{-3}	5.8786	1.7713	0.8448		
(b)	108	motor		0.6554				
		operated;		1.5073			3.3567	2.20
(c)	56.3	ball	1.33×10^{-3}	1.33×10^{-2}	-4.3200	0.8448		

Cutsets 8 through 14 are numerically the same as cutset 7. Table 16 combines the lognormal distributions in Table 15, using the reproductive theorem; see equations 5 and 6.

TABLE 16. PARAMETER VALUES FOR CASE I-GAMMA

Term Number	Lognormal Distribution		Gamma Distribution	
	μ	σ	λ	p+1
P_I -A	1.7713	0.8448		
-B			3.3567	2.20
P_{II} -A	0.8901	0.8448		
-B			3.3567	2.20
P_{III} -A	0.2852	0.8448		
-B			3.3567	2.20
P_{IV} -A			3.3567	2.20
-B			3.3567	2.20
P_V -A			3.3567	2.20
-B			3.3567	2.20
P_{VI} -A	-1.0626	1.1947		
-B			3.3567	2.20
P_{VII} -A	-2.5487	1.1947		
-B			3.3567	2.20

Terms VIII through XIV have the same parameter values as Term VII.

$$P = 10^{-8} \times \sum_{i=I}^{i=XIV} P_{i-A} \times P_{i-B}$$

Table 16 lists the parameter values for Case I-gamma (μ, σ for the lognormal and λ, p+1 for the gamma distributions.).

Figure 7 gives the probability density for Case I-gamma. Figure 8 gives the CDF for this case. The factor 10^{-8} is introduced to convert the abscissa values to probabilities. See Appendix 1 for the methodology of the calculation. A probability vs. percentile graph of Case I-gamma is presented in Figure 6, permitting a comparison with Case I (Design Option B-2).

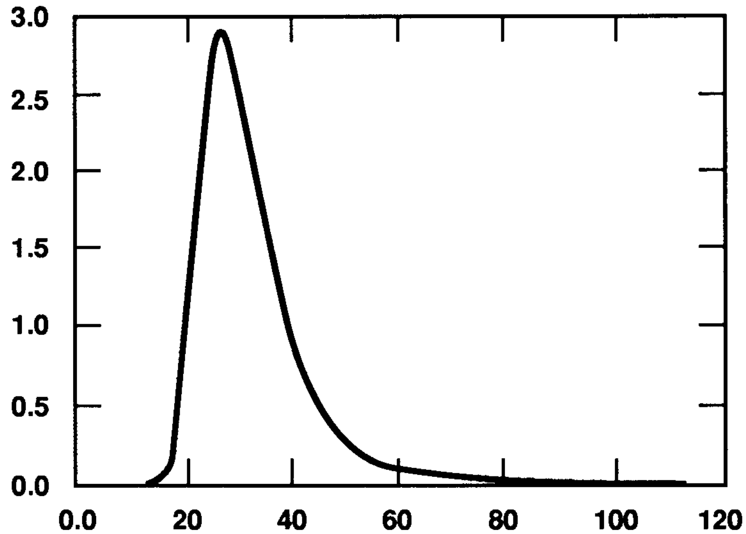

Figure 7. Density function in arbitrary units for Case I-gamma.

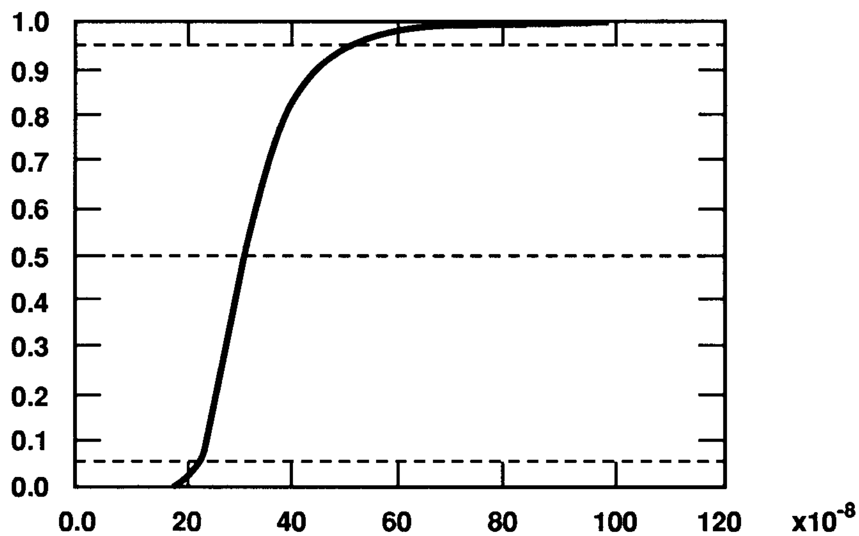

Figure 8. Cumulative distribution function for Case I-gamma. The factor 10^{-8} is introduced to convert abscissa values to probabilities.

Density and CDF for probability of failure of brake system

Fig 7&8

It was stated earlier that Cases I and I-gamma differed in two respects. The point estimate failure rate was changed for the motor operated valve from 6.0×10^{-8} to 3.2×10^{-8} ("overall"), and the distribution from lognormal to a two parameter gamma distribution. In order to complete the picture, a calculation was also made (not described in this report) of Case I-R in which the point estimate value was changed, but not in the distribution, which remained lognormal.

Table 17 lists the defining parameters and the values of the probabilities at mean and 95th percentiles in order to permit comparisons.

TABLE 17. COMPARISON OF CASES I; I-R; I-GAMMA

Case	Motor Operated Valve		Ball Valve		Mean ($\times10^{+6}$)	95 percentile ($\times10^{+6}$)
	distribution	point est	distribution	point est		
I	lognormal	6.0×10^{-8}	lognormal	0.65×10^{-6}	0.34	0.81
I-R	lognormal	3.2×10^{-8}	lognormal	0.65×10^{-6}	0.12	0.27
I-gamma	gamma	3.2×10^{-8}	lognormal	0.65×10^{-6}	0.32	0.51

Table 17 indicates that diminishing the initial point estimate value for the motor operated valve by a factor of approximately 2 will cause a reduction in the mean and 95 percentile values of the probabilities by a factor of approximately 3. Similarly a change in distribution only from lognormal to gamma for the motor operated valve brings a factor of increase of approximately 2 (95 percentile) to 3 (mean value). See Figure 6.

CALCULATIONS FOR CASE V-R

Case V-R uses data for the motor operated valves derived from the LERs, Table 14, NUREG/CR-2770, for R_1 (M-1). As in the calculation for Case I-gamma, a two parameter gamma distribution is used to describe the failure data. The two data items used will be the mean and 95 percentile values for the failure rates.

$$\text{mean} = 4.8\times10^{-7}$$
$$\text{95 perc.} = 1.4\times10^{-6}$$

The same methodology described previously is now used to obtain p and λ. Assume an annual work load of 2,048 hours.

$$\text{mean} = 4.8\times10^{-7}\times2{,}048 = 0.9830\times10^{-3}$$
$$\text{95 perc.} = 1.4\times10^{-6}\times2{,}048 = 0.2867\times10^{-2}$$

Using a normalization factor of 10^3:

$$\bar{x} = 0.9830\times10^{-3}\times10^{3} = 0.9830$$
$$x_{95} = 0.2867\times10^{-2}\times10^{3} = 2.867$$

$$\frac{x_{95}}{\bar{x}} = \frac{2.867}{0.9830} = \frac{U_{95}}{\sqrt{p+1}} = 2.9166$$

From Pearson's tables p = 0.081; p+1=1.081

$$\text{Also,}\lambda = \frac{p+1}{\bar{x}} = \frac{1.081}{0.9830} = 1.0997$$

One may now set out Table 18 which is the equivalent of Table 15 for this case, retaining the EP values for the globe and ball valves.

TABLE 18. CALCULATIONS FOR CASE V-R

Cutset No.	Valve No.	Valve Type	EP	10^6×EP	μ	σ	λ	p+1
1(a)	25.2.4	globe	5.8786×10^{-4}	0.58786	-0.5313	0.8448		
(b)	108	motor		0.9830				
		operated		2.867			1.0997	1.081
2(a)	25.1.2	globe	2.4354×10^{-4}	0.24354	-1.4125	0.8448		
	25.3.4							
(b)	108	motor		0.9830				
		operated		2.867			1.0997	1.081
3(a)	56.3.6	ball	1.33×10^{-4}	0.133	-2.0174	0.8448		
(b)	108	motor		0.9830				
		operated		2.867			1.0997	1.081
4(a)	51	motor		0.9830				
		operated		2.867			1.0997	1.081
(b)	108	motor		0.9830				
		operated		2.867			1.0997	1.081
5(a)	45	motor		0.9830				
		operated		2.867			1.0997	1.081
(b)	108	motor		0.9830				
		operated		2.867			1.0997	1.081
6(a)	25.4	globe	5.8786×10^{-3}	5.8786	1.7713	0.8448		
(b)	108	motor		0.9830				
		operated		2.867			1.0997	1.081
(c)	25.2	globe	5.8786×10^{-3}	5.8786×10^{-3}	-5.1364	0.8448		
7(a)	25.4	globe	5.8786×10^{-3}	5.8786	1.7713	0.8448		
(b)	108	motor		0.9830				
		operated		2.867			1.0997	1.081
(c)	56.3	ball	1.33×10^{-3}	1.33×10^{-3}	-6.6226	0.8448		

Table 19 gives the final (μ, σ) and (λ, p+1) values for the cutsets, using the reproductive theorem to combine the lognormal distributions in cutsets 6 and 7, Table 18. Note that the difference in EP values for the ball and globe valves (3a, 7c; 1a, 2a, 6a and 7a) by a factor of 10 arises from common cause failures occurring in cutsets (1a, 2a) for the globe valves, and (3a) for the ball valves.

TABLE 19. PARAMETER VALUES FOR CASE V-R

Cutset Number	Lognormal Distribution		Gamma Distribution	
	μ	σ	λ	p+1
P_I -A	-0.5313	0.8448		
-B			1.0997	1.081
P_{II} -A	-1.4125	0.8448		
-B			1.0997	1.081
P_{III} -A	-2.0174	0.8448		
-B			1.0997	1.081
P_{IV} -A			1.0997	1.081
-B			1.0997	1.081
P_V -A			1.0997	1.081
-B			1.0997	1.081
P_{VI} -A	-3.3651	1.1947		
-B			1.0997	1.081
P_{VII} -A	-4.8513	1.1947		
-B			1.0997	1.081

Terms VIII through XIV have the same parameter values as Term VII.

$$P = 10^{-6} \times \sum_{i=I}^{i=XIV} P_{i-A} \times P_{i-B}$$

Figures 9 and 10 give the probability density and the CDF for Case V-R. Abscissa values are multiplied by 10^{-6} to produce probabilities. See Appendix 1 for the methodology of the calculation. In order to make a comparison with Case V-R, a calculation was made (details not included in this report) for Case V-lambda (+), using the data from Table 14 for the motor operated valves. We also decided to make a calculation similar to case V-R, but to base it on the use of the 5% confidence value and the 95% confidence value (Table 14).

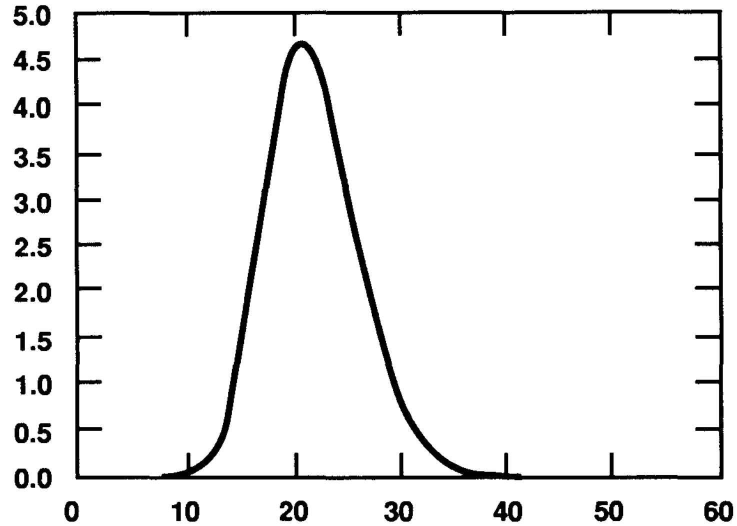

Figure 9. Density function in arbitrary units for Case V-R.

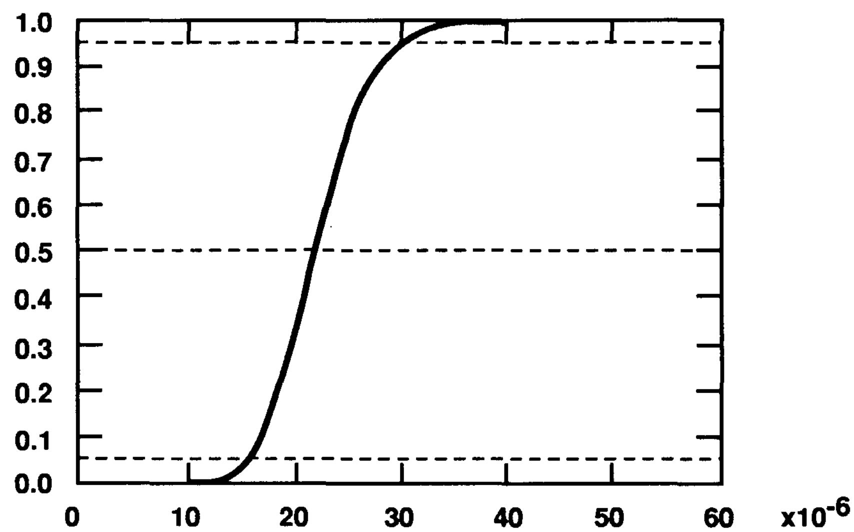

Figure 10. Cumulative distribution function for Case V-R. The factor 10^{-6} is introduced to convert abscissa values to probabilities.

Density and CDF for probability of failure of brake system

The purpose was to learn whether the results would be similar to the calculation based on the use of the mean and 95% confidence values. The "5%, 95%" calculation is called Case V (details not included in this report). Table 20 compares the results for Cases V, V-R and V-lambda(+). The data for Case I-gamma is also included, since it permits a calculation of the sensitivity with which probability values depend on the initial point estimate failure rates. See Figure 11 for graphs of V, V-R and V-lambda(+) for the cumulative distribution functions for probability of failure of the brake system.

TABLE 20. COMPARISON OF CASES V, V-R, V-LAMBDA(+), I-GAMMA

Failure Data for Motor Operated Valves from Tables 14, 15. Cumulative Distribution Functions(CDF)							
Case	Point Estimate Failure Rates (1/hr)					Probability $\times 10^6$	Probability $\times 10^6$
	5% Confidence	Mean	95% Confidence	λ	p+1	Mean	95 perc
V	1.5×10^{-7}		1.4×10^{-6}	1.963	2.562	25.6	31.9
V-R		4.8×10^{-7}	1.4×10^{-6}	1.0997	1.081	22.0	29.6
V-lambda(+)		3.6×10^{-7}	7.9×10^{-7}	3.4975	2.5787	15.6	19.6
I-gamma		3.2×10^{-8}	7.36×10^{-8}	3.3567	2.20	0.32	0.51

The values for the probability, both mean and 95 percentile, for the two Cases V and V-R, are very similar, differing by only 10% to 15%. An analysis, based on Cases V-R, V-lambda(+) and I-gamma, of the scale effect of increasing the mean failure rate of the motor operated valves indicates a power law dependence for the probability, P values:

Approximately, $(P)_{95\ perc} \sim (\text{mean point estimate failure rate})^{1.5}$

$(P)_{mean} \sim (\text{mean point estimate failure rate})^{1.6}$

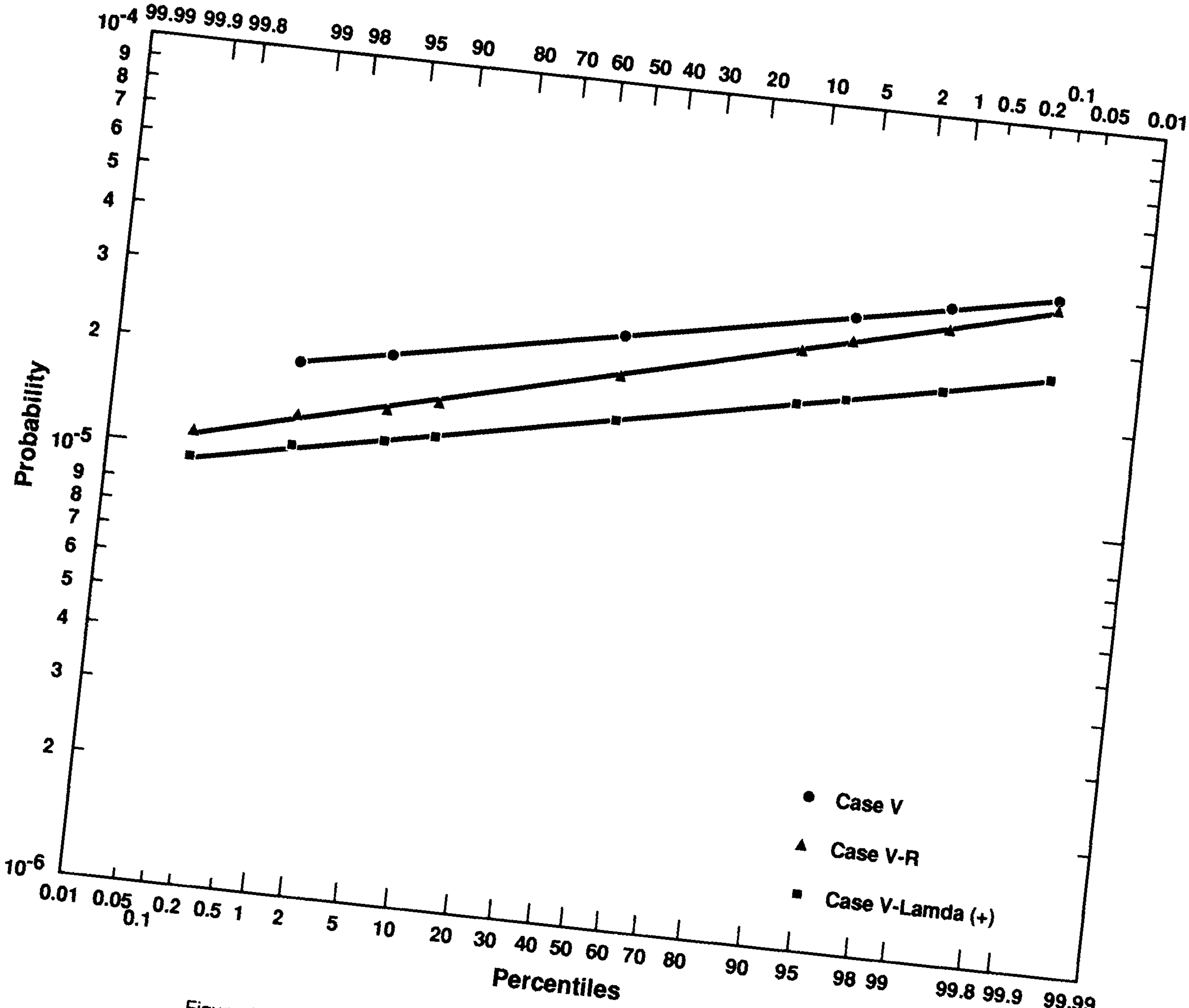

Figure 11. Cumulative distribution functions for probability of failure of brake system.

The faster than linear increase in probability (P) values as a function of the mean failure rate for motor operated valves can be understood by looking at Tables 15 and 18. The sensitivity of the P values results from the presence of the parameters for the motor operated valves in each term, and doubly in cutsets 4 and 5.

IX. VALUES OF PARAMETERS FOR MANUAL BALL VALVES

The failure rate for manual ball valves used in the DOE reports is taken from IEEE Std 500-1984, p. 1044, reference 4: Nonelectric Parts Reliability data, NPRD-2 (Reliability Analysis Center 1981), summer 1981; see Table 8; the listed value is 0.65×10^{-6} (1/hr). This report has been superseded by Nonelectronic Parts Reliability Data, NPRD-3, 1985 (Rossi 1985). A portion of the relevant data, page 150, is given in Table 21.

TABLE 21. FAILURE RATES FOR BALL VALVES PER MILLION HRS.

Series	Point Estimate	20% Lower Interval	80% Upper Interval	# of Records	# of Failures	Operating Hours(10^6)
A	9.861	7.820	12.456	5	17	1.724
B	0.647	0.400	1.029	2	5	7.723

As the table indicates one series corresponds to the NPRD-2 (Reliability Analysis Center 1981) value of 0.65×10^{-6}1/hr. However, an additional series of data produces the failure rate of 9.861×10^{-6}, a factor of 15 greater. The authors state that the data can be represented by an exponential distribution, a special case of the two parameter gamma distribution. Actually, as will be shown later in this report, both distributions produce similar results in the present case. The point estimates (means) are calculated as the ratio of the number of failures to the number of operating hours.

In Chapter VIII, calculations of CDFs were based on data from Licensee Event Reports for motor operated valves. In those calculations, the data for manual ball valves were based on the IEEE older reference NPRD-2 (Reliability Analysis Center 1981) (see Table 8). The distribution used with those data was lognormal. In this chapter, data for the manual ball valves will be taken from the more recent NPRD-3 (Rossi 1985), and will include both the A and B series (see Table 21).

CALCULATIONS FOR CASE VII-B

This case is similar to Cases V-R and V except for the change from a lognormal to a two parameter gamma distribution for the manual ball valves.

From Table 21:

The point estimate (mean) failure rate for series B is 0.647×10^{-6}(1/hr). Call the 20% lower interval and the 80% upper interval failure rates Q_{20} and Q_{80}.

$$Q_{20} = 0.400\times10^{-6}(1/hr)$$
$$Q_{80} = 1.029\times10^{-6}(1/hr)$$

Then the event probabilities per annum (EP) are:

$$EP_{20} = Q_{20}\times2,048(hrs) = 0.8192\times10^{-3}$$
$$EP_{80} = Q_{80}\times2,048(hrs) = 2.1074\times10^{-3}$$

Use 10^3 as a multiplier to obtain "x" values:

$$x_{20} = 10^3(EP)_{20} = 0.8192$$
$$x_{80} = 10^3(EP)_{80} = 2.1074$$

$$\text{The ratio} \quad \frac{x_{80}}{x_{20}} = \frac{u_{80}}{u_{20}} = \frac{2.1074}{0.8192} = 2.572 = 2.57$$

From Pearson's Tables one finds:

$$p = 2.5$$

$$u_{20} = 1.0211$$

$$u_{80} = 2.6207$$

$$\frac{u_{80}}{u_{20}} = 2.566 = 2.57$$

Thus $$\lambda = \frac{u\sqrt{p+1}}{x}$$

$$= \frac{u_{20}\sqrt{p+1}}{x_{20}} = \frac{1.0211\sqrt{3.5}}{0.8192} = 2.332$$

Also $$\lambda = \frac{u_{80}\sqrt{p+1}}{x_{80}} = \frac{2.6207\sqrt{3.5}}{2.1074} = 2.326$$

Let $$\lambda = 2.33$$

One may now set out parameter values in Table 22 for this case, retaining the EP values for the globe valve (Table 18), and the (λ, p+1) values for the motor operated valves (Table 20, Case V).

TABLE 22. CALCULATIONS FOR CASE VII-B

Cutset No.	Valve No.	Valve Type	EP	10^6 EP	μ	σ	λ	p+1
1(a)	25.2.4	globe	5.8786×10^{-4}	0.58786	-0.5312	0.8448		
(b)		motor		0.3072				
		operated		2.867			1.963	2.562
2(a)	25.1.2	globe	2.4354×10^{-4}	0.24354	-1.4126	0.8448		
	25.3.4							
(b)	108	motor		0.3072				
		operated		2.867			1.963	2.562
3(a)	56.3.6	ball	0.8192×10^{-4}	0.8192×10^{-1}			23.3	3.5
			2.1074×10^{-4}	2.1074×10^{-1}				
(b)	108	motor		0.3072				
		operated		2.867			1.963	2.562
4(a)	51	motor		0.3072				
		operated		2.867			1.963	2.562
(b)	108	motor		0.3072				
		operated		2.867			1.963	2.562
5(a)	45	motor		0.3072				
		operated		2.867			1.963	2.562
(b)	108	motor		0.3072				
		operated		2.867			1.963	2.562
6(a)	25.4	globe	5.8786×10^{-3}	5.8786	1.7713	0.8448		
(b)	108	motor		0.3072				
		operated		2.867			1.963	2.562
(c)	25.2	globe	5.8786×10^{-3}	5.8786×10^{-3}	-5.1364	0.8448		
7(a)	25.4	globe	5.8786×10^{-3}	5.8786×10^{-3}	-5.1364	0.8448		
(b)	108	motor		0.3072				
		operated		2.867			1.963	2.562
(c)	56.3	ball	0.8192×10^{-3}	0.8192				
			2.1074×10^{-3}	2.1074			2.33	3.5

Table 23 gives the (μ, σ) and (λ, p+1) values for the cutsets, using the reproductive theorem in cutset 6. Cutsets 8 through 14 are identical numerically to cutset 7.

TABLE 23. PARAMETER VALUES FOR CASE VII-B

Number	μ	σ	λ	p+1
P_I -A -B	-0.5312	0.8448	 1.963	 2.562
P_{II} -A -B	-1.4126	0.8448	 1.963	 2.562
P_{III} -A -B			23.3 1.963	3.5 2.562
P_{IV} -A -B			1.963 1.963	2.562 2.562
P_V -A -B			1.963 1.963	2.562 2.562
P_{VI} -A -B	-3.3651	1.1947	 1.963	 2.562
P_{VII} -A -B -C	-5.1364	0.8448	 1.963 2.33	 2.562 3.5

Terms VIII through XIV have the same parameter values as Term VII.

$$P = 10^{-6} \times \left[\sum_{i=I}^{i=VI} P_{i\text{-}A} \times P_{i\text{-}B} + \sum_{i=VII}^{i=XIV} P_{i\text{-}A} \times P_{i\text{-}B} \times P_{i\text{-}C} \right]$$

Figures 12 and 13 give the probability density and the CDF for Case VII-B. Abscissa values are multiplied by 10^{-6} to obtain values for P. See Appendix 1 for the methodology of the calculation. Table 24 gives a comparison of selected P values for Case VII-B with Case V.

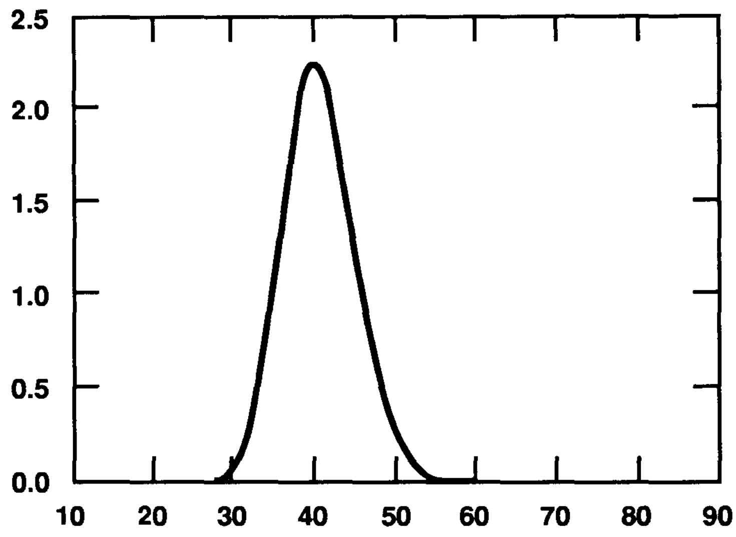

Figure 12. Density function in arbitrary units for Case VII-B.

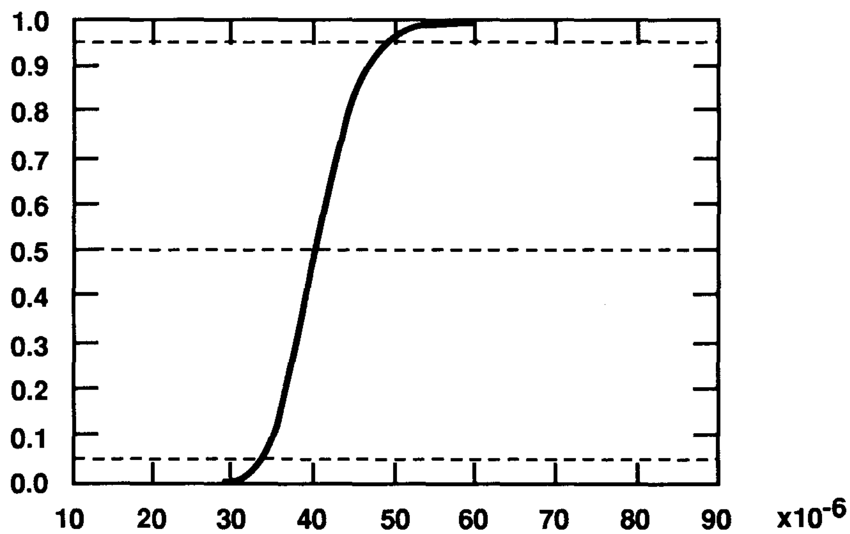

Figure 13. Cumulative distribution function for Case VII-B. The factor 10^{-6} is introduced to convert abscissa values to probabilities. The calculated mean is 40.65×10^{-6}. The calculated quantiles: .01, .05, .1, .5, .9, .95, .99 are (31.02, 33.60, 35.05, 40.48, 46.5, 48.33, 51.98) $\times 10^{-6}$ respectively.

Density and CDF for probability of failure of brake system

TABLE 24. COMPARISON OF CASES V, VII-B

Case	Motor operated valves Point estimates (Table 15)	Ball Valves Distribution	$P \times 10^6$ mean	$P \times 10^6$ 95 perc
V	(5 perc) 1.5×10^{-7} (95 perc) 14×10^{-7}	lognormal	25.6	31.9
VII-B	(5 perc)1.5×10^{-7} (95 perc) 14×10^{-7}	two parameter gamma	40.7	48.3

The change in distribution for the ball values, going from lognormal to a two parameter gamma distribution, produced a factor of increase in P values of the mean of approximately 1.6. Figure 14 shows the graphs for Cases V and VII-B for the cumulative distribution functions for probability of failure of the brake system.

CALCULATIONS FOR CASE IX-B

Case IX-B is the same as Case VII-B with but a single change. Use is made of an exponential rather than a two parameter gamma distribution. The frequency curve for the two parameter gamma distribution is given by equation (7):

$$f(x) = \frac{\lambda(\lambda x)^p \, e^{-\lambda x}}{\Gamma(p+1)} \qquad (7)$$

The exponential distribution is a special case in which p=0. Equation (7) becomes:

$$f(x) = \lambda e^{-\lambda x} \qquad (11)$$

Also the mean value, $\bar{x}$, is given by:

$$\bar{x} = \frac{1}{\lambda} \qquad (12)$$

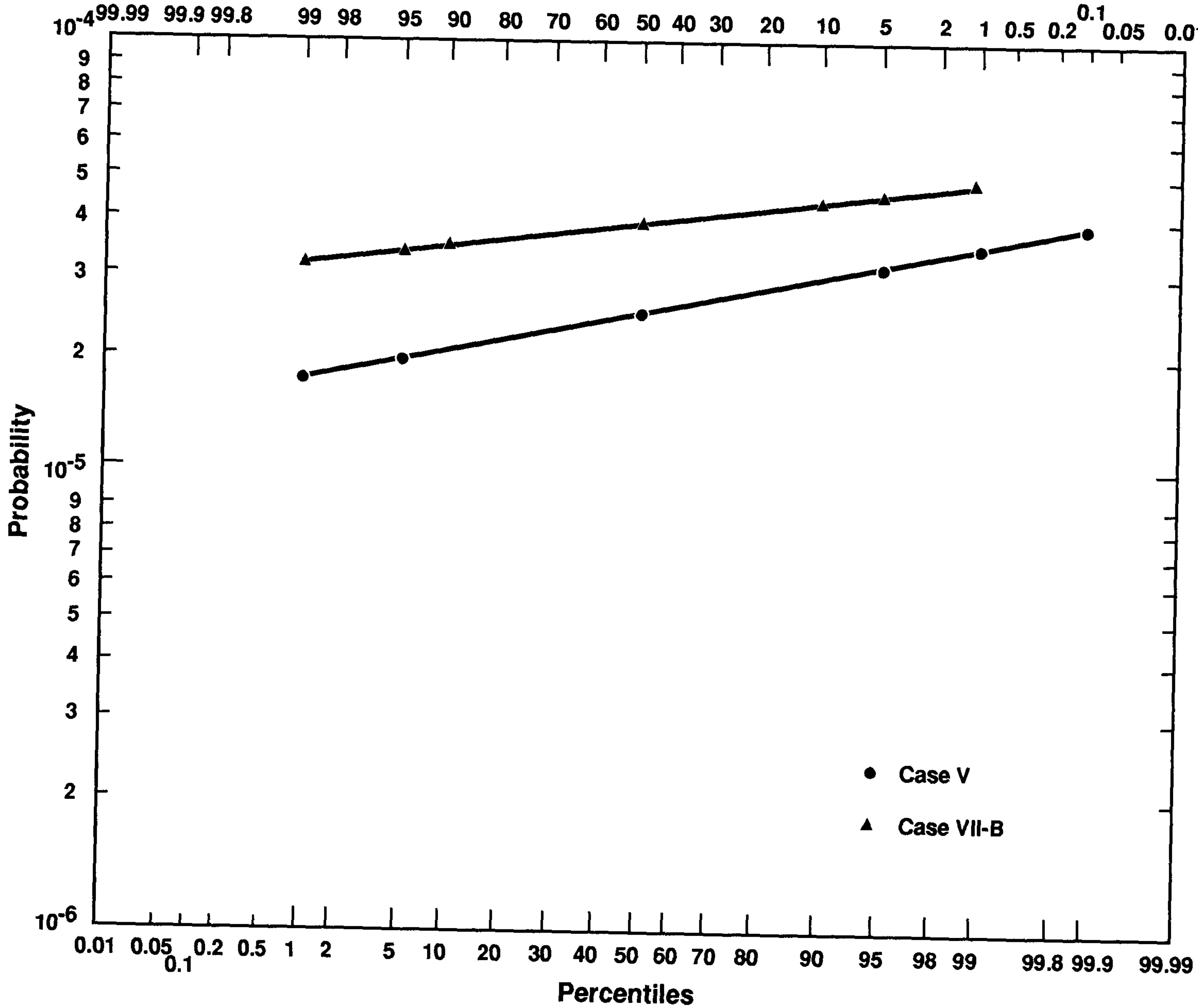

Figure 14. Cumulative distribution functions for probability of failure of brake system.

Conversely $$\lambda = \frac{1}{\bar{x}} \qquad (13)$$

Thus the exponential distribution has but a single parameter, λ, which is simply related to the mean value. In Case VII-B the two parameters chosen were the values of the 20% lower interval, and the 80% upper interval from Table 21. In Case IX-B the single parameter chosen is the point estimate for the mean value = 0.647×10^{-6}(1/hr); Table 21. Using an annual work load of 2,048 hours:

$$(EP) = 0.647\times10^{-6}\times2{,}048$$
$$= 1.325\times10^{-3}$$

Introduce a factor of 10^3 to obtain the "$\bar{x}$" value.

$$10^3\times(EP) = \bar{x} = 1.325$$

From equation (13) for λ:

$$\lambda = \frac{1}{1.325} = 0.7547$$

Table 25 gives the parameter values for this case. The only changes from Table 23 are the (λ, p) values for the ball valves, which occur in terms III-A, and VII-C. See Tables 22 and 23.

Figures 15 and 16 give the probability density and the CDF for Case IX-B. Table 26 gives a comparison between Cases V, VII-B and IX-B. See Appendix 1 for the methodology of the calculation.

TABLE 25. PARAMETER VALUES FOR CASE IX-B

Number	μ	σ	λ	p+1
P_I -A	-0.5312	0.8448		
-B			1.963	2.562
P_{II} -A	-1.4126	0.8448		
-B			1.963	2.562
P_{III} -A			7.547	1.0
-B			1.963	2.562
P_{IV} -A			1.963	2.562
-B			1.963	2.562
P_V -A			1.963	2.562
-B			1.963	2.562
P_{VI} -A	-3.3652	1.1947		
-B			1.963	2.562
P_{VII} -A	-5.1365	0.8448		
-B			1.963	2.562
-C			0.7547	1.0

Terms VIII through XIV are numerically the same as Term VII (A, B, C).

$$P = 10^{-6} \left[\sum_{i=I}^{i=VI} P_{i-A} \times P_{i-B} + \sum_{i=VII}^{i=XIV} P_{i-A} \times P_{i-B} \times P_{i-C} \right]$$

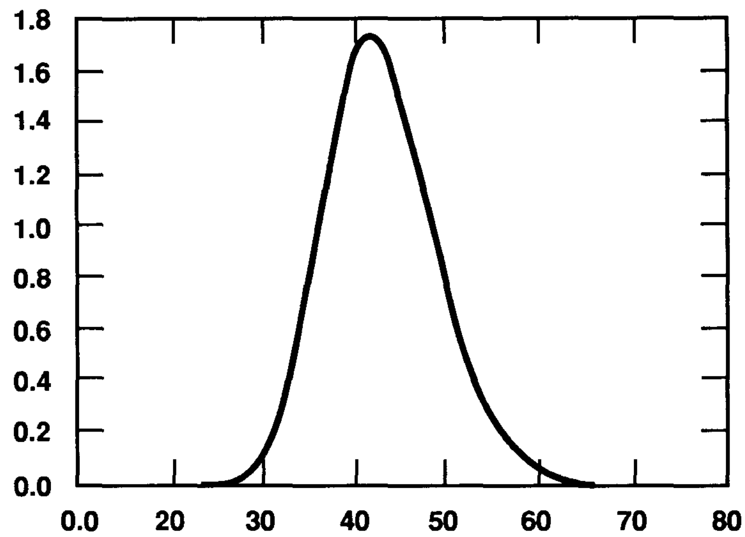

Figure 15. Density function in arbitrary units for Case IX-B.

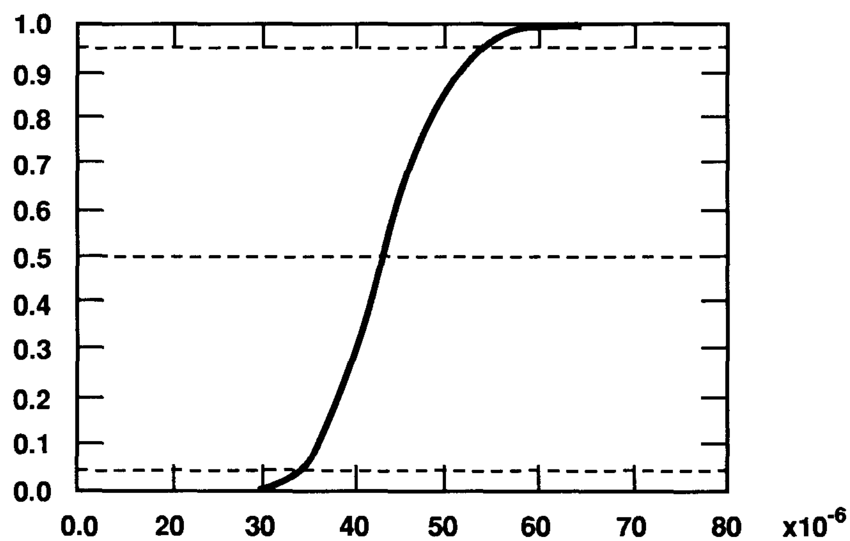

Figure 16. Cumulative distribution function for Case IX-B. The factor 10^{-6} is introduced to convert abscissa values to probabilities. The calculated mean is 42.80 x 10^{-6}. The calculated quantiles: .01, .05, .1, .5, 0.9, .95, .99 are (30.60, 33.78, 35.55, 42.48, 50.45, 52.95, 57.9) x 10^{-6} respectively.

Density and CDF for probability of failure of brake system

Fig 15&16

TABLE 26. COMPARISON OF CASES V, VII-B, IX-B

Case	Motor Operated Valves: Percentile Values (Table 14)	Ball Valves: Table 21 Distribution, Mean or Percentile Values	$P\times10^6$ Mean	$P\times10^6$ 95 perc.
V	(5 perc) 1.5×10^{-7} (95 perc) 14×10^{-7}	lognormal (mean) 0.647×10^{-6}	25.6	31.9
VII-B	(5 perc) 1.5×10^{-7} (95 perc) 14×10^{-7}	two parameter gamma (20 perc) 0.400×10^{-6} (80 perc) 1.029×10^{-6}	40.7	48.3
IX-B	(5 perc) 1.5×10^{-7} (95 perc) 14×10^{-7}	one parameter exponential (mean) 0.647×10^{-6}	42.8	52.9

Table 26 shows that the P values for Cases VII-B and IX-B are within 5% (mean) to 10% (95 perc) of each other.

CALCULATIONS FOR CASE VII-A

This case is similar to Case VII-B except that it is based on Series A data, Table 21. The calculations follow the same format used for Case VII-B. A two parameter gamma distribution is used based on the 20% and 80% interval data.

$$Q_{20} = 7.820\times10^{-6}(1/hr)$$

$$Q_{80} = 12.456\times10^{-6}(1/hr)$$

$$EP_{20} = Q_{20}\times2,048 \text{ (hrs)} = 1.6015\times10^{-2}$$

$$EP_{80} = Q_{80}\times2,048 \text{ (hrs)} = 2.5510\times10^{-2}$$

Use 10^3 as a multiplier to obtain x values.

$$x_{20} = 10^3(EP)_{20} = 16.015$$

$$x_{80} = 10^3(EP)_{80} = 25.510$$

$$\text{The ratio, } R = \frac{25.510}{16.015} = 1.593$$

From Pearson's Tables: $p = 12.4$; $p+1 = 13.4$

Also $u_{20} = 2.8035$; $u_{80} = 4.4659$

$$\lambda = \frac{u\sqrt{p+1}}{x} = \frac{2.8035}{16.015}\sqrt{13.4}$$

$$\lambda = 0.6408$$

One may now set out Table 27, similar to Table 23 except for the (λ, p+1) values for Case VII-A, which appear in terms P_{III-A} and P_{VII-C} for ball valves.

TABLE 27. PARAMETER VALUES FOR CASE VII-A

Number	μ	σ	λ	p+1
P_I -A	-0.5312	0.8448		
-B			1.963	2.562
P_{II} -A	-1.4126	0.8448		
-B			1.963	2.562
P_{III} -A			6.408	13.4
-B			1.963	2.562
P_{IV} -A			1.963	2.562
-B			1.963	2.562
P_V -A			1.963	2.562
-B			1.963	2.562
P_{VI} -A	-3.3651	1.1947		
-B			1.963	2.562
P_{VII} -A	-5.1364	0.8448		
-B			1.963	2.562
-C			0.6408	13.4

Terms VIII through XIV are identical numerically to VII (A, B, C).

$$P = 10^{-6}\left[\sum_{i=I}^{i=VI} P_{i-A} \times P_{i-B} + \sum_{i=VII}^{i=XIV} P_{i-A} \times P_{i-B} \times P_{i-C}\right]$$

Figures 17 and 18 give the probability density and the CDF for Case VII-A. See Appendix 1 for the methodology of calculation. Table 28 compares selected P values for Cases VII-B and VII-A.

TABLE 28. COMPARISON OF CASES VII-B, VII-A

Case	Ball Valves: Table 21 Distribution Percentile Values	$P\times10^6$ Mean	$P\times10^6$ 95 perc.
VII-B	two parameter gamma (20 perc) 0.400×10^{-6} (80 perc) 1.029×10^{-6}	40.7	48.3
VII-A	two parameter gamma (20 perc) 7.820×10^{-6} (80 perc) 12.456×10^{-6}	199	227

Note that although the 20 percentile and 80 percentile failure rates increase by factors of 12 to 20, the P values increase only by a factor of 5. This is in contrast to the situation for the motor operated valves, for which the P values had a power dependence on the input failure rates. The less sensitive dependence, in the case of the ball valves, is a result of fewer terms in which ball valve failure rates appear. See Table 22. The first term in which the ball valve failure rate appears is 3(a), followed by 7(c), 8(c), etc.

CALCULATIONS FOR CASE IX-A

Case IX-A is similar to Case IX-B in that a single parameter exponential distribution is used, based however on Series A data, Table 21. The format used in Case IX-B is followed. The mean failure rate is:

$$Q_A = 9.861\times10^{-6}(1/hr)$$

$$(EP)_A = Q_A\times2{,}048 \text{ hrs.} = 9.861\times10^{-6}\times2{,}048$$

$$(EP)_A = 20.195\times10^{-3}$$

$$\bar{x} = 10^3(EP)_A = 20.195$$

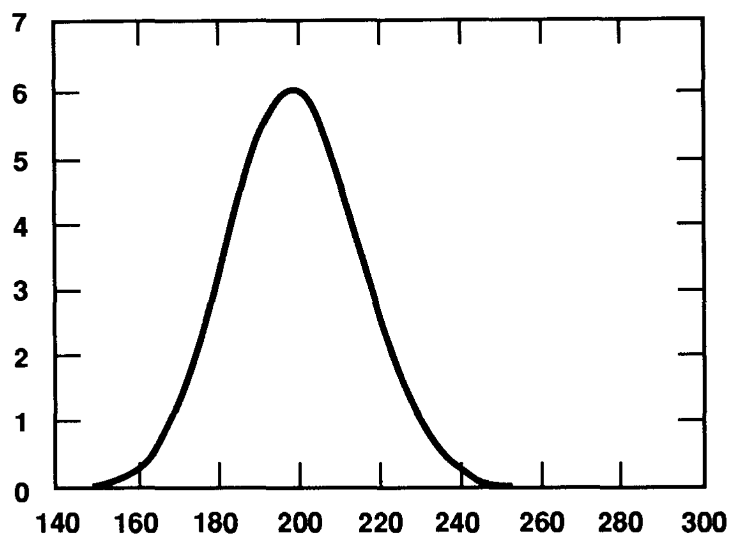

Figure 17. Density function in arbitrary units for Case VII-A.

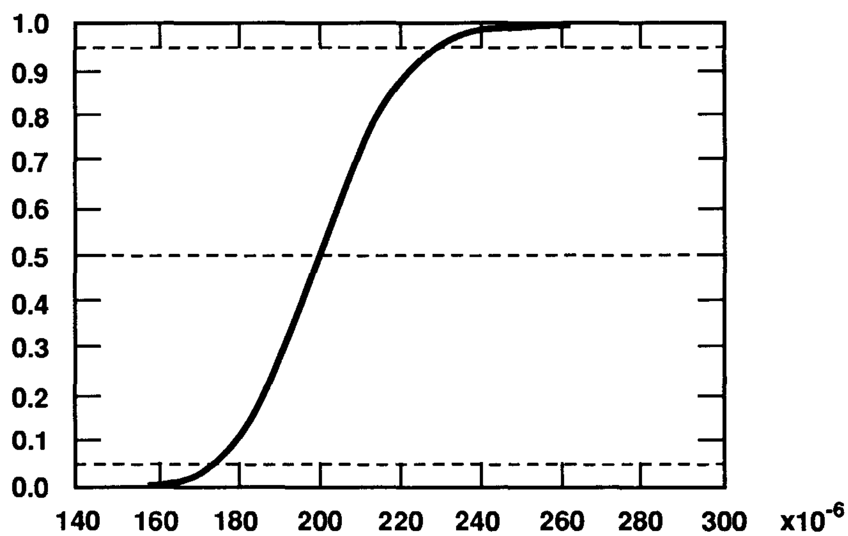

Figure 18. Cumulative distribution function for Case VII-A. The factor 10^{-6} is introduced to convert abscissa values to probabilities. The calculated mean is199.1 x 10^{-6}. The calculated quantiles: .01, .05, .1, .5, 0.9, .95, .99 are (162.5, 172.6, 178.1, 198.6, 220.7, 227.3, 240.0) x 10^{-6} respectively.

$$\lambda = \frac{1}{\bar{x}} = \frac{1}{20.195} = 0.04952$$

$$p = 0$$

Table 29 sets out the parameter values which are similar to Table 27, except for the (λ, p+1) values for the ball valve, series A data, terms III-A, VII-C.

TABLE 29. PARAMETER VALUES FOR CASE IX - A

Number	μ	σ	λ	p+1
P_I -A -B	-0.5312	0.8448	 1.963	 2.562
P_{II} -A -B	-1.4126	0.8448	 1.963	 2.562
P_{III} -A -B			0.4952 1.963	1.0 2.562
P_{IV} -A -B			1.963 1.963	2.562 2.562
P_V -A -B			1.963 1.963	2.562 2.562
P_{VI} -A -B	-3.3651	1.1947	 1.963	 2.562
P_{VII} -A -B -C	-5.1364	0.8448	 1.963 0.04952	 2.562 1.0

Terms P_{VIII} through P_{XIV} are identical numerically to $P_{VII\ (A,B,C)}$.

$$P = 10^{-6} \left[\sum_{i=I}^{i=VI} P_{i-A} \times P_{i-B} + \sum_{i=VII}^{i=XIV} P_{i-A} \times P_{i-B} \times P_{i-C} \right]$$

Figures 19 and 20 give the probability density and the CDF for Case IX-A. See Appendix 1 for the methodology of calculation. Table 30 compares selected P values for Cases IX-B, IX-A and VII-A.

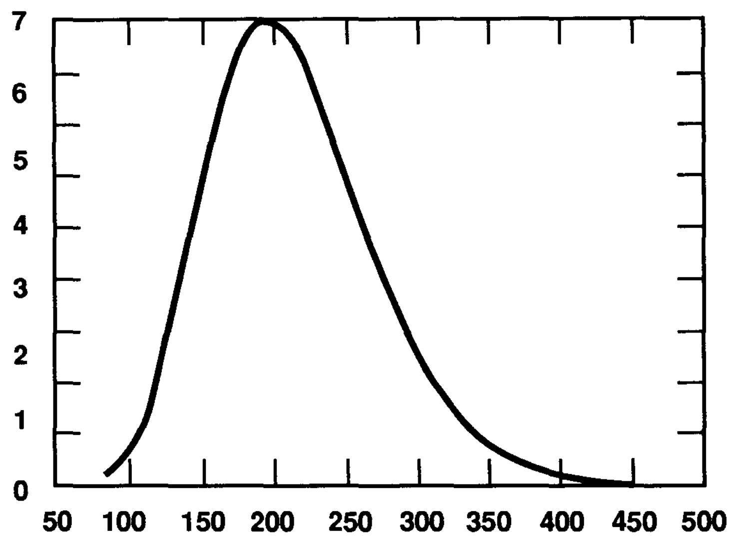

Figure 19. Density function in arbitrary units for Case IX-A.

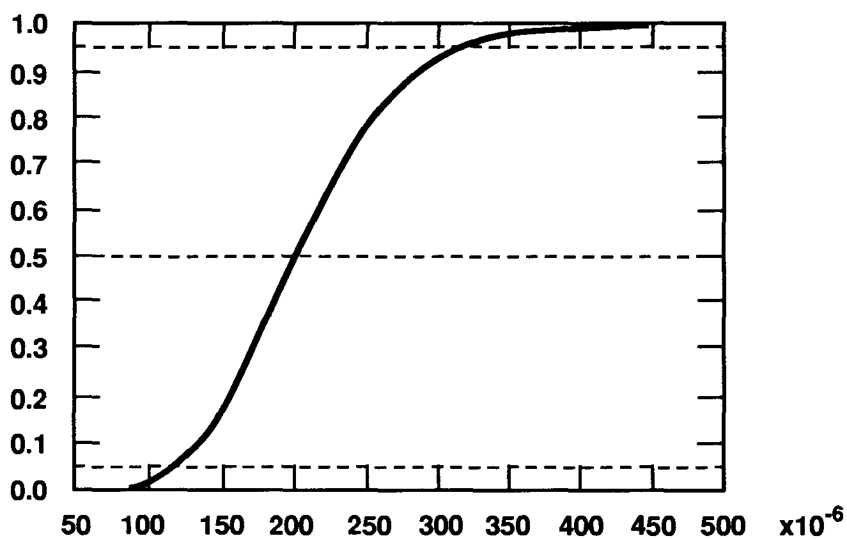

Figure 20. Cumulative distribution function for Case IX-A. The factor 10^{-6} is introduced to convert abscissa values to probabilities. The calculated mean is 214.7 x 10^{-6}. The calculated quantiles: .01, .05, .1, .5, 0.9, .95, .99 are (106.3, 130.0, 144.5, 208.2, 293.1, 321.4, 379.7) x 10^{-6} respectively.

Density and CDF for probability of failure of brake system

Fig 19&20

TABLE 30. COMPARISON OF CASES IX-B, IX-A, VII-A

Case	Ball Valves: Table 21 Distribution, Mean Failure Rate	$P \times 10^6$ Mean	$P \times 10^6$ 95 perc
IX -B	one parameter exponential 0.647×10^{-6}	42.8	52.9
IX -A	one parameter exponential 9.861×10^{-6}	215	321
VII -A	two parameter gamma (20 perc) 7.820×10^{-6} (80 perc) 12.456×10^{-6}	199	227

The increases in P for Case IX-A, compared to Case IX-B are by factors of 5 and 6, approximately, whereas the increase in the mean failure rate was by a factor of 15. This is similar to what occurred in Cases VII-A and VII-B. Comparing Case VII-A with Case IX-A, changing from a two parameter gamma distribution to an exponential distribution, increases the P by 8% (mean) and 41% (95 percentile). Figure 21 compares cases IX-B, IX-A, and VII-A graphically for the cumulative distribution functions for probability of failure of the brake system.

X. CALCULATIONS OF BOUNDING CASES

The calculations in Chapter IX complete the utilization of the basic data provided by the sources listed in Table 8. It is now appropriate to consider the possibilities of calculating reasonable bounding cases, given the nature of the data available. One may consider the situation for the motor driven valves, and separately for the manual ball valves.

The discussion of the sources of the data (LERs) in NUREG/CR-1363 and NUREG/CR-2770 (see Chapter VII and Table 8) suggests that a lower bound set of values for the motor driven valves be chosen that were used in the

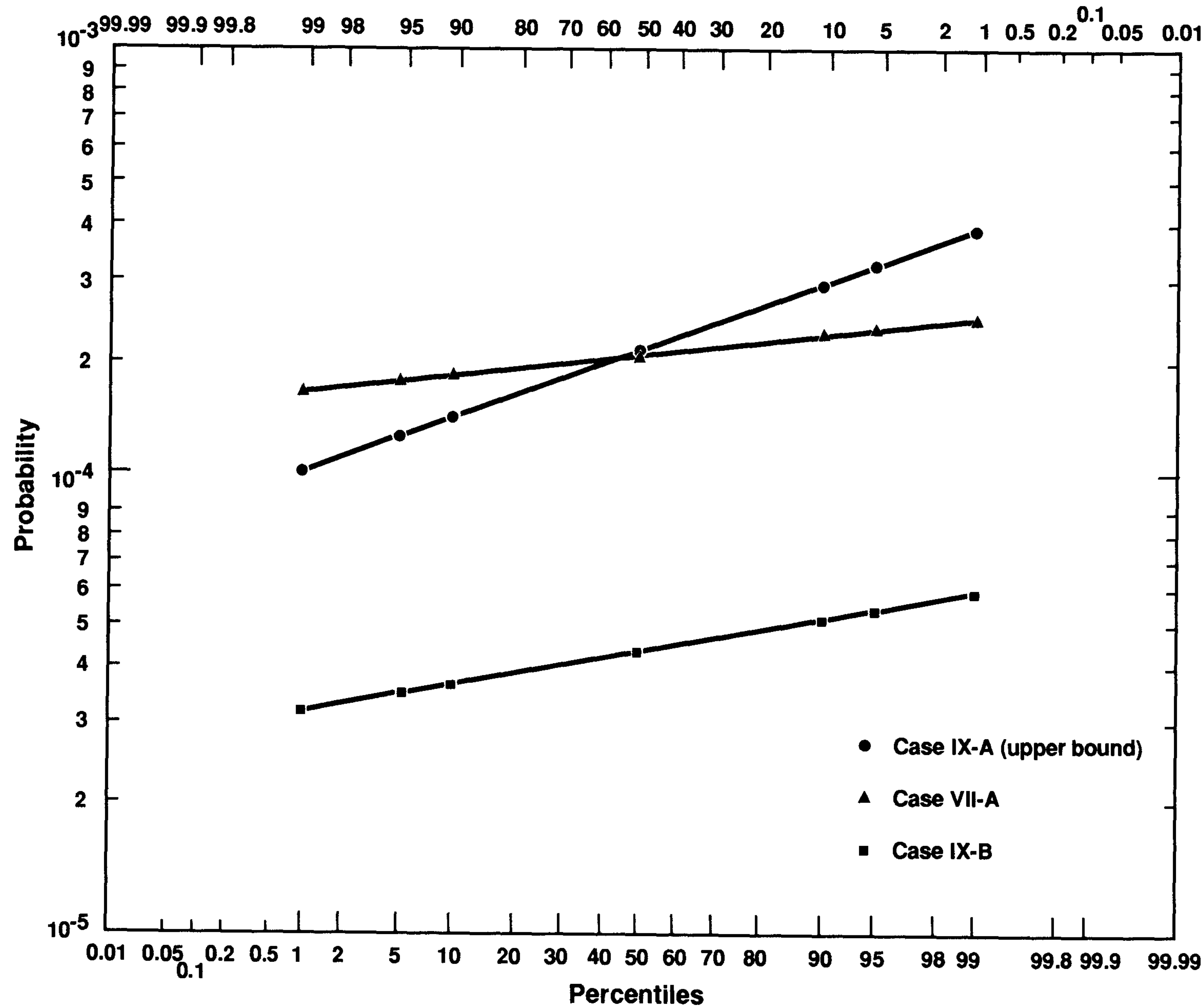

Figure 21. Cumulative distribution functions for probability of failure of brake system.

calculations for Case I-gamma (see Tables 15, 16, 17), the "overall" data in Table 13 with mean failure rate of 3.2×10^{-8}. Similarly an upper bound set of values for the motor driven valves were chosen and used in Case V (see Table 14 with mean failure of 4.8×10^{-7}, also Table 20).

The situation for the manual ball valves differs from that of the motor driven valves. As Table 21 indicates, there are two distinct data series, A and B, with point estimates differing by more than an order of magnitude (~15). This suggests there may not exist just one simple generic type of manual ball valve. Without additional information about the manual ball valves actually in place in the waste hoist, prudence suggests making the assumption that series A manual ball valves are in place. Thus it is proposed to use the "A" data for the manual ball valve in both the lower and upper bound calculations. Actually, the calculations already performed in Case IX-A have utilized "A" data for the manual ball valves (exponential distribution), and Case V data for the motor driven valves. Thus Case IX-A will serve as the "upper bound" case.

For the lower bound case the data suggested above for the motor driven valve taken from Case I-gamma will be combined with the "A" data for the manual ball valve (with exponential distribution). This case is named "Case IX-A:I-gamma." Table 31 lists the calculations for this case. Note that the EP values for the globe valves are the same that appear in Table 15. The EP values for the motor operated valves are the same as those in Table 15 for the Case I-gamma. The λ, p+1 values for the motor operated valves are also the same in both Tables 15 and 31.

Table 32 sets out the parameter values for the calculation of Case IX-A:I-gamma. Figures 22 and 23 give the probability density and the CDF for Case IX-A:I-gamma. Figure 24 shows the graphs of the cumulative distribution functions for probability of failure of the brake system. These graphs represent the lower bound, Case IX-A:I-gamma, and the upper bound, represented by Case IX-A, already depicted in Figure 21. Table 33 summarizes the parameters used in the various cases that were calculated.

TABLE 31. CALCULATIONS FOR CASE IX-A:I-GAMMA

Cutset No.	Valve No.	Valve Type	EP	10^8 EP ($\bar{x}$)	μ	σ	λ	p+1
1(a)	25.2.4	globe;	5.8786×10^{-4}	5.8786	1.7713	0.8448		
(b)	108	motor	0.6554×10^{-4}	0.6554				
		operated	1.5073×10^{-4}	1.5073			3.3567	2.2
2(a)	25.1.2	globe;	2.4354×10^{-4}	2.4354	0.8901	0.8448		
	25.3.4							
(b)	108	motor		0.6554				
		operated		1.5073			3.3567	2.2
3(a)	56.3.6	ball;	20.195×10^{-4}	20.195			0.04952	1.0
(b)	108	motor		0.6554				
		operated		1.5073			3.3567	2.2
4(a)	51	motor		0.6554				
		operated;		1.5073			3.3567	2.2
(b)	108	motor		0.6554				
		operated		1.5073			3.3567	2.2
5(a)	45	motor		0.6554				
		operated;		1.5073			3.3567	2.2
(b)	108	motor		0.6554				
		operated		1.5073			3.3567	2.2
6(a)	25.4	globe;	5.8786×10^{-3}	5.8786	1.7713	0.8448		
(b)	108	motor		0.6554				
		operated;		1.5073			3.3567	2.2
(c)	25.2	globe	5.8786×10^{-3}	5.8786×10^{-2}	-2.8339	0.8448		
7(a)	25.4	globe;	5.8786×10^{-3}	5.8786×10^{-2}	-2.8339	0.8448		
(b)	108	motor		0.6554				
		operated;		1.5073			3.3567	2.2
(c)	56.3	ball	20.195×10^{-3}	20.195			0.04952	1.0

TABLE 32. PARAMETER VALUES FOR CASE IX-A:I-GAMMA

Term Number	Lognormal Distribution		Gamma Distribution	
	μ	σ	$b = \lambda$	$K = p+1$
P_I -A	1.7713	0.8448		
-B			3.3567	2.2
P_{II} -A	0.8901	0.8448		
-B			3.3567	2.2
P_{III} -A			0.04952	1.0
-B			3.3567	2.2
P_{IV} -A			3.3567	2.2
-B			3.3567	2.2
P_V -A			3.3567	2.2
-B			3.3567	2.2
P_{VI} -A	-1.0626	1.1947		
-B			3.3567	2.2
P_{VII} -A	-2.8339	0.8448		
-B			3.3567	2.2
-C			0.04952	1.0

Terms P_{VIII} through P_{XIV} are identical numerically to $P_{VII(A,B,C)}$.

$$P = 10^{-8} \times \left[\sum_{i=I}^{i=VI} P_{i\text{-}A} \times P_{i\text{-}B} + \sum_{i=VII}^{i=XIV} P_{i\text{-}A} \times P_{i\text{-}B} \times P_{i\text{-}C} \right]$$

TABLE 33. SUMMARY OF PARAMETER VALUES AND DISTRIBUTIONS FOR SELECTED CASES

Case	Manual Ball Valves	Motor Operated Valves
	Distribution, Failure Rate, 1/hr	Distribution, Failure Rate, 1/hr
I (Design Option B-2)	lognormal med: 0.65×10^{-6}	lognormal med: 6.0×10^{-8}
I-R	lognormal med: 0.65×10^{-6}	lognormal med: 3.2×10^{-8}
I-gamma	lognormal med: 0.65×10^{-6}	2 parameter gamma mean: 3.2×10^{-8}
V	lognormal med: 0.65×10^{-6}	2 parameter gamma 5 percentile: 1.5×10^{-7} 95 percentile: 1.4×10^{-6}
V-R	lognormal med: 0.65×10^{-6}	2 parameter gamma mean: 4.8×10^{-7} 95 percentile: 1.4×10^{-6}
VII-A	2 parameter gamma 20 percentile: 7.82×10^{-6} 80 percentile: 12.46×10^{-6}	2 parameter gamma 5 percentile: 1.5×10^{-7} 95 percentile: 1.4×10^{-6}
VII-B	2 parameter gamma 20 percentile: 0.40×10^{-6} 80 percentile: 1.029×10^{-6}	2 parameter gamma 5 percentile: 1.5×10^{-7} 95 percentile: 1.4×10^{-6}
IX-A	exponential mean: 9.861×10^{-6}	2 parameter gamma 5 percentile: 1.5×10^{-7} 95 percentile: 1.4×10^{-6}
IX-B	exponential mean: 0.647×10^{-6}	2 parameter gamma 5 percentile: 1.5×10^{-7} 95 percentile: 1.4×10^{-6}
IX-A:I-gamma	exponential mean: 9.861×10^{-6}	2 parameter gamma mean: 3.2×10^{-8}

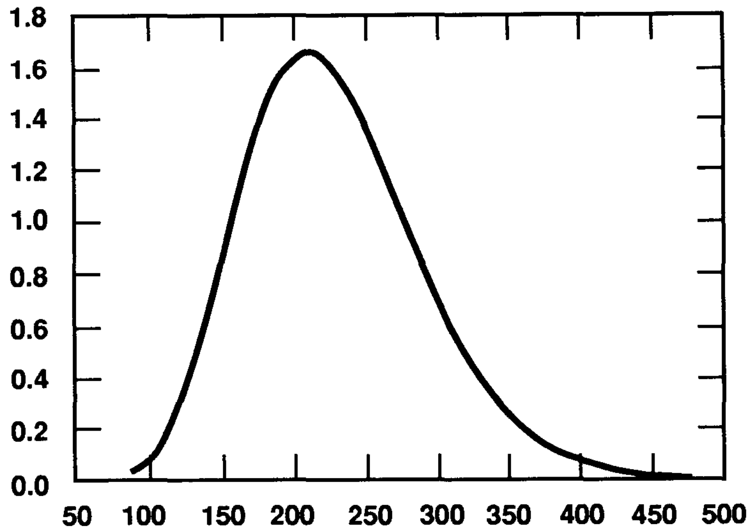

Figure 22. Density function in arbitrary units for Case IX-A: l-gamma.

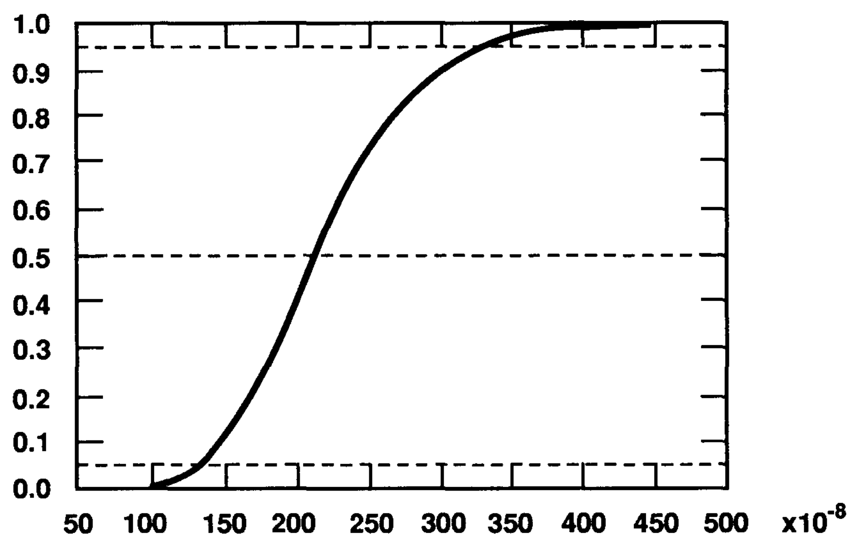

Figure 23. Cumulative distribution function for Case IX-A: l-gamma. The factor 10^{-8} is introduced to convert abscissa values to probabilities. The calculated mean is 229.6 x 10^{-8}. The calculated quantiles: .01, .05, .1, .5, 0.9, .95, .99 are (113.1, 139.0, 154.9, 223.1, 312.8, 342.4, 403.3) x 10^{-8} respectively.

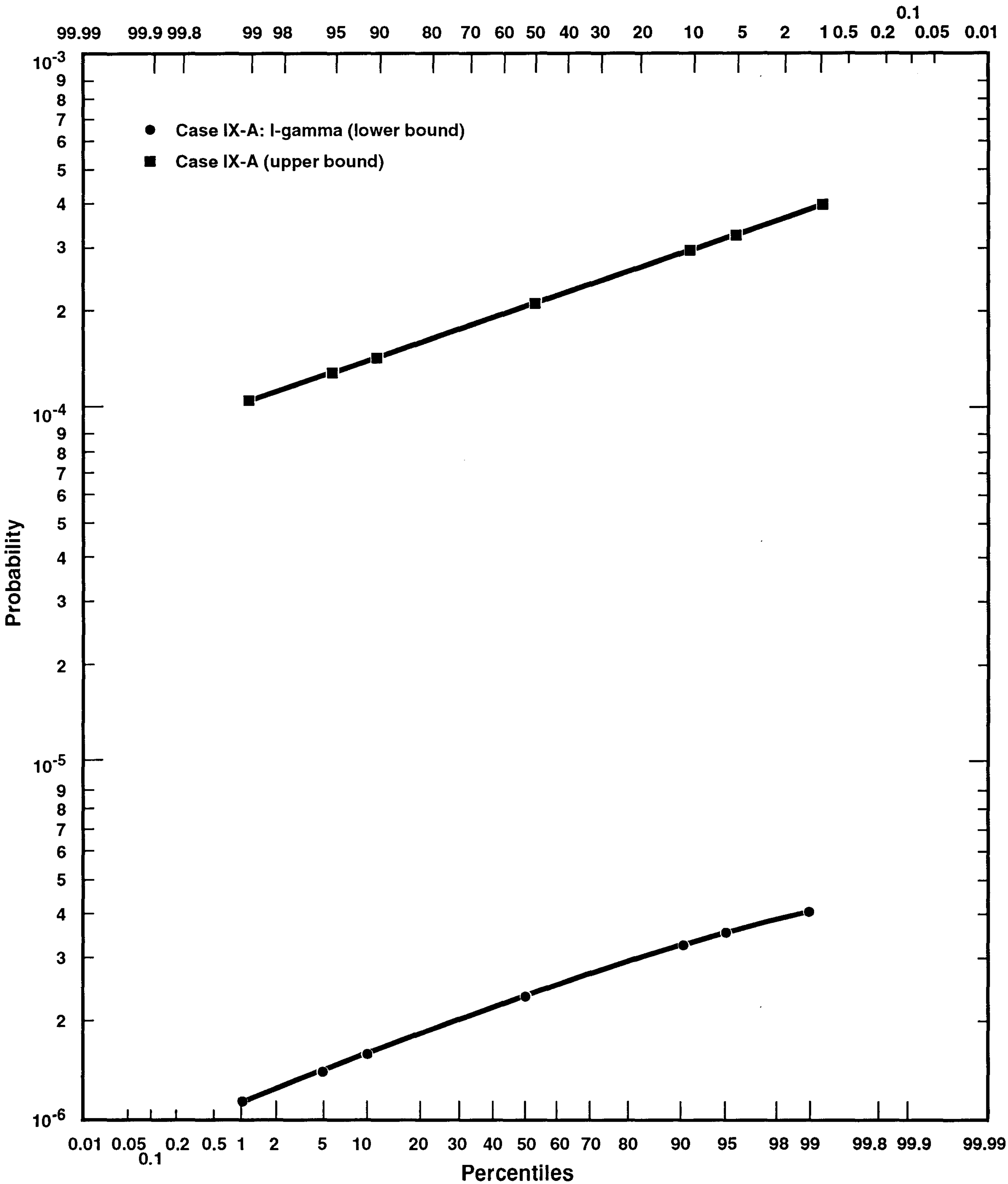

Figure 24. Cumulative distribution function for probability of failure of brake system.

XI. DISCUSSION

The analysis in this report is based on failure data for the various types of valves used in the waste hoist. The data sources are listed in Table 8, and include the references used by DOE in their IRA DOE/WIPP-89-010, 1990 and the follow up references cited in this report. It may be noted that the data for the motor driven valves, a crucial type in this study, are based on Licensee Event Reports (LERs) extending only through 1980, some 13 years ago at the time of this writing. A second important type of valve, the manual ball valves, has two distinct sets of failure data which differ by more than an order of magnitude. This information was published in 1985 in the Nonelectronic Parts Reliability Data (NPRD-3). A question to be considered, raised earlier in this report, is whether there exists more than one generic type of manual ball valve.

A discussion of the nature of the data for the motor driven values which appear in NUREG/CR-2770 (see Table 8) emphasizes imperfections in the data which are obtained from Licensee Event Reports. One consequence is the considerable range of possible failure rates listed in Table 14, derived from NUREG/CR-1363 and NUREG/CR-2770. Both reports are based on the same basic data sets, the LERs. For this reason, this report chose to calculate a lower and upper bound, using the values for the failure rates listed in Table 14. An additional choice that is required is the selection of a failure rate for the manual ball valves, listed in Table 21. There are two distinct sets of data, the A series and the B series. Without additional information, prudence dictates selection of the higher failure rate, the A series, to be used in the bounding calculations.

The failure data were then utilized in the calculation of the probability density and cumulative distribution function for a random variable, the probability of failure of the brake system. The methodology for these calculations is described in the appendices of this report.

The bounding cases described above are given in Figure 24, (lower bound, Case IX-A:I-gamma and upper bound, Case IX-A). For the lower bound, the probability of failure varies from approximately 10^{-6} (1 percentile) to

4×10^{-6} (99 percentile) per year. For the upper bound, the probability of failure varies from approximately 10^{-4} (1 percentile) to 4×10^{-4} (99 percentile) per year, a full two orders of magnitude greater than the lower bound.

Clearly there is a need to lower this large range between the bounding calculations. This is possible only with the acquisition of more data to reduce or eliminate the ambiguities and questions discussed in this report. In Chapter III of this report on "Use of Confidence Intervals vs. Point Estimates Alone" the quotation from the NRC in the Federal Register emphasized the importance of performing sensitivity studies "to determine those uncertainties most important to the probabilistic estimates." The analyses in this report make it clear that the failure rates for the motor driven valves are the most important in determining the probabilistic estimates. More information in this area may help narrow the difference in the bounding calculations.

Finally one may note that the use of point estimates alone may produce values of probability of failure that are not conservative.

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APPENDICES

APPENDIX 1

SUMMARY OF METHODS USED IN CALCULATIONS

Our building blocks are numerical methods to compute probability distributions of two types of random variables. The first is a random variable that is the *sum* of two independent random variables. The second is a random variable that is the *product* of two independent random variables. To compute the distribution of a *sum* of two independent random variables, we use Fast Fourier Transforms to implement the calculus of characteristics functions.[1] To compute the distribution of a *product* of two independent random variables, we exploit the additional fact that the log of a product is the sum of the logs.

Transform Methods

The heart of the calculation is a method for computing the density function for a random variable that is the *sum* of two independently distributed random variables with known densities.[2] To compute the distribution of any such sum, we use the following theorems.

Theorem: Let x be a continuously distributed random variable with density f(x), and let y be a continuously distributed random variable with density g(y). Let x and y be independently distributed. Then the random variable $z = x + y$ is distributed with density h(z) given by the *convolution* of f and g, which is defined by

$$h(z) = \int f(u)g(z - u)du$$

There is a discrete-random variable version of this theorem, which governs discrete approximations to continuously distributed random variables. Here it is.

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[1] That is, we use transform methods to implement the property that the density of the sum of two independent random variables is the convolution of their densities.

[2] The mathematical theorems we are quoting can be found in many books on operational mathematics. For example, see R. A. Gabel and R. A. Roberts, *Signals and Linear Systems*, Wiley, 1973.

Theorem: Let x be a discretely distributed random variable that takes values on the set $X = [x_0, x_1, \ldots, x_{T-1}]$, with density given by $f_t = \text{Prob}[x = x_t]$. Let y be a discretely distributed random variable that takes values on the *same* set X, with density given by $g_t = \text{Prob}[y = x_t]$. Let z be the discretely distributed random variable $z = x + y$, and let x and y be distributed independently. Then z has density h determined by[3]

$$h_t = \sum_k f_k g_{t-k}$$

where $h_t = \text{Prob}[z = z_t]$, and where z resides in the discrete set $Z = [2x_0, \ldots, 2x_{T-1}]$.

The next useful result is the fact that the Fourier transform of a convolution is the *product* of the Fourier transforms of the two sequences being convoluted. The *Fourier transform* of a sequence $\{x_t\}_{t=0}^{t=T-1}$ is defined as the sequence of complex numbers $x(w_j)$ given by

$$x(w_j) = \sum_{t=0}^{T-1} x_t e^{-iw_j t} \tag{1}$$

where $w_j = 2\pi j/T$ and $j = 0, 1, \ldots, T-1$. The *inverse Fourier transform* is given by

$$x_t = T^{-1} \sum_{j=0}^{T-1} x(w_j) e^{iw_j t} \tag{2}$$

Equations (1) and (2) constitute the basic Fourier transform pair. Notice that the inverse Fourier transform of the Fourier transform is the original sequence.

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[3] For Sensitivity Case I, we directly computed the convolution. For all other cases, we used the Fast Fourier Transform methods.

Now here is the key theorem we use:

Theorem: The Fourier transform of the convolution of two sequences $\{x_t\}$ and $\{y_t\}$ is the *product* of their Fourier transforms $x(w_j)$, $y(w_j)$.

We apply this theorem as follows. For each of two continuous distributions, (f,g), the probability laws for (x,y), respectively, we put down a discrete 'grid' of points $X = [x_0, \ldots, x_{T-1}]$ on the real line, with the points spaced close enough together and over a sufficiently large set to approximate each continuous distribution well. Then we used (f,g) to generate approximating discrete probability distribution for (x,y). For computational consistency, we used the *same* grid for each random variable under study. We chose the grid carefully to make sure that each random variable as well as the relevant sums were well approximated by the procedure. For each approximating distribution $\hat{f}_t$ and $\hat{g}_t$, we computed the Fourier transform $f(w_j)$ and $g(w_j)$. Then we computed the Fourier transform of $\{\hat{h}_t\}$, the approximating distribution of the sum of $x + y$, as

$$h(w_j) = f(w_j)g(w_j)$$

Notice how this uses the preceding theorem. To compute the approximate density of $x + y$, $\hat{h}_t$, we then inverse Fourier transformed $h(w_j)$:

$$\hat{h}_t = T^{-1} \sum_{t=0}^{T-1} h(w_j)e^{iw_j t}$$

<u>Computational Details</u>

We implemented these calculations using the *Fast Fourier Transform* (FFT) and the associated inverse transform, the IFFT. We used the computer language MATLAB on a SUN Sparc-2 Workstation. This permitted us to put down very large and fine grids. We used one (inconsequential) approximation: each time a convolution is computed, the FFT in effect *truncates* the grid on which the relevant sum is distributed, and restricts it to the same domain on which the original two distributions are defined.[4] In particular, the density of the sum is computed only on the *same* domain $X = [x_0, x_1, \ldots, x_{T-1}]$, rather than on the true domain $Z = [2x_0, \ldots, 2x_{T-1}]$. To control the error resulting from this approximation, we select the grid set X very carefully to make sure that it covers the region where the pertinent x,y, and sum $z = x + y$ have appreciable positive probability.

<u>Distribution of a Product</u>

To compute the distribution of the product of two independent random variables, we use the fact that any functions of independent random variables are also independent random variables. Let X be a random variable with density $\tilde{f}(X)$. Let $Y = \tilde{g}(X)$ be a monotone transformation of X. Define $dY/dX = \tilde{g}'(x) \neq 0$, by monotonicity. Define the inverse function $X = \tilde{g}^{-1}(Y)$, where $d\tilde{g}^{-1}(Y)/dY = dX/dY = 1/\tilde{g}'(X)$. Then Y has the density[5]

$$\tilde{f}_Y(Y) = \tilde{f}(X)\,|dX/dY| \tag{3}$$

Our first step is to apply (3) where g is the natural log function. In this case, we get

$$\tilde{f}_Y(Y) = \tilde{f}(X)X \tag{4}$$

To calculate the density of $x_{1i} \cdot x_{2i}$, we compute the densities of the log of x_{1i} and the log of x_{2i}, then calculate the Fast Fourier transform of these densities, multiply the Fourier transforms, then inverse Fourier transform to get the density of the product of the logs.

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[4] This is the cost of using the FFT. The cost is more than compensated for in terms of the size of the grids that we can handle swiftly. We experimented with calculating convolutions directly, and actually used this method for Case I. That is, we convoluted the sequences f,g to get h using $h_t = \sum_k f_k g_{t-k}$. This proved to be very time-intensive. We achieved accurate results much more quickly using the transform methods. Also, please note that the results that we are reporting are vastly more accurate than would be calculations using Monte Carlo methods, controlling for computer time.

[5] For example, see Bernard Lindren, *Statistical Theory*, Third Edition, MacMillan Publishers, 1976; p. 454.

For the final step, we apply (3) once more, this time using the exponential function as the $\tilde{g}$ function, to calculate the density of the exponential of the log of $x_{1i} \times x_{2i}$, which of course is simply the density of $x_{1i} \times x_{2i}$. The appropriate version of (3) for the exponential function $\tilde{g}$ is simply

$$\alpha_Z(Z) = h_z(\log(Z))/Z \tag{5}$$

where h_z is the density of the sum of the logs, and α is what we want, namely, the density of the product $x_{1i}x_{2i}$.

APPENDIX 2

COMPUTER PROGRAMS

The computer programs used to calculate the probability distributions are available from Professor Thomas Sargent. They can be obtained through electronic mail by sending a request to sargent@riffle.stanford.edu. Alternatively write to Professor Thomas Sargent, Hoover Institution, Stanford, California 94305.